THE REASSEMBLING OF SHATTERED BROWNIAN SHEET

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If $D_n : n \geq 1$ is a given countable collection of Borel subsets of the unit square $[0,1]^2 = I^2$ and Z is a standard Brownian sheet over I^2, is it possible to reconstruct Z from the knowledge of all of the patches $Z - c_n$ over transformed domains $\tau_n(D_n)$ for unknown constants c_n and unknown rotation-translation transformations τ_n? We show that the answer to this question is yes under fairly natural restrictions on the sets D_n. The main property of Brownian sheet that leads to this possibility is that the local behaviour of Z around a point $\mathbf{t}$ actually determines $\mathbf{t}$. In this sense, a Brownian sheet carries with it its own location coordinates.

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1 Introduction

It is a privilege to be able to contribute to this volume in honor of Professor van Zwet on the occasion of his 65^{th} birthday. I have known Bill for about half of those 65 years, giving me many opportunities to observe and benefit from his warm hospitality, clever insights, wise counsel and keen enthusiasm evidenced throughout his numerous theoretical and professional undertakings.

In this written contribution I consider a specific question about Brownian sheet, one that might imply that if Humpty-Dumpty were to have had a 'Brownian complexion' then the ending to the popular nursery rhyme may have concluded with "All the king's men and all the king's horses *could* put Humpty together again."

I believe it is fair to say that most mathematical researchers work primarily on problems that are interrelated and part of long term programs. However, most of us also enjoy the challenging diversions that come along in the form of easily stated, fairly specific, open questions, especially when they do not easily yield to available theory and techniques. When pursued, such problems can have the beneficial result of leading the pursuers to new methods and theory. Such was the nature of the question mentioned by R. M. Dudley at a 1976 Oberwolfach meeting: "Does the Kakutani interval-splitting procedure (in which at each stage the largest spacing is uniformly