AN EXPONENTIAL INEQUALITY FOR A WEIGHTED APPROXIMATION TO THE UNIFORM EMPIRICAL PROCESS WITH APPLICATIONS

DAVID M. MASON[1]

University of Delaware

Mason and van Zwet (1987) obtained a refinement to the Komlós, Major, and Tusnády (1975) Brownian bridge approximation to the uniform empirical process. From this they derived a weighted approximation to this process, which has shown itself to have some important applications in large sample theory. We will show that their refinement, in fact, leads to a much stronger result, which should be even more useful than their original weighted approximation. We demonstrate its potential applications through several interesting examples. These include a useful new exponential inequality for Winsorized sums and results on the asymptotic equivalence of two sequences of local experiments.

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1 Introduction and statements of main results

Let $U, U_1, U_2, \ldots$, be independent uniform $(0,1)$ random variables. For each integer $n \geq 1$ let

$$G_n(t) = n^{-1} \sum_{i=1}^{n} 1\{U_i \leq t\}, \quad -\infty < t < \infty, \tag{1}$$

denote the *empirical distribution function* based on $U_1, \ldots, U_n$, and

$$\alpha_n(t) = \sqrt{n}\{G_n(t) - t\}, \ 0 \leq t \leq 1, \tag{2}$$

be the corresponding *uniform empirical process*. Mason and van Zwet (1987) proved the following refinement to the Komlós, Major, and Tusnády [KMT] (1975) Brownian bridge approximation to α_n.

Theorem 1.1 *There exists a probability space $(\Omega, \mathcal{A}, P)$ with independent uniform $(0,1)$ random variables $U_1, U_2, \ldots$, and a sequence of Brownian bridges $B_1, B_2, \ldots$, such that for all $n \geq 1$, $1 \leq d \leq n$ and $x \in \mathbb{R}$*

$$P\left\{ \sup_{0 \leq t \leq d/n} |\alpha_n(t) - B_n(t)| \geq n^{-1/2}(a \log d + x) \right\} \leq b \exp(-cx) \tag{3}$$

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