# ON CENTRAL LIMIT THEORY FOR RANDOM ADDITIVE FUNCTIONS UNDER WEAK DEPENDENCE RESTRICTIONS

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Random additive functions defined on intervals provide a general framework for varied applications including (dependent) array sums, and level-exceedance measures for stochastic sequences and processes. Central limit theory is developed in Leadbetter and Rootzén (1993) for families $\{\zeta_T(I) : T > 0\}$ of such functions under (array forms of) standard strong mixing conditions. One objective of the present paper is to introduce a potentially much weaker and more readily verifiable form of strong mixing under which the limiting distributional results are shown to apply. These lead to characterization of possible limits for such $\zeta_T(I)$ as those for independent array sums, i.e. the classical infinitely divisible types. The conditions and results obtained for one interval are then extended to apply to joint distributions of $\{\zeta_T(I_j) : 1 \le j \le p\}$ of (disjoint) intervals $I_1, I_2, \ldots I_p$, asymptotic independence of the components being shown under the extended conditions. Similar results are shown under even slightly weaker conditions for positive, additive families. Under *countable* additivity this leads in particular to distributional convergence of random measures under these mixing conditions, to infinitely divisible random measure limits having independent increments.

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## 1 Introduction

By a random additive function (r.a.f.) we mean a random function $\zeta(I)$ defined for subintervals $I = (a, b]$ of the unit interval and additive in the sense that $\zeta(a, b] + \zeta(b, c] = \zeta(a, c]$ when $0 < a < b < c \le 1$. To our knowledge, such a framework was first used in proving a central limit theorem in the early (and pioneering) paper Volkonski and Rozanov (1959).

As described in Leadbetter and Rootzén (1993) and (1997), r.a.f. *families* $\{\zeta_T(I)\}$ (or $\{\zeta_n(I)\}$) provide a simple unifying framework for (array) central limit problems for both discrete and continuous parameter processes. This includes the general limiting distributional properties of array sums and exceedance measures, which are useful in a variety of areas such as environmental regulation and structural reliability (cf. Leadbetter and Huang (1996)). For example with obvious notation, for $I = (a, b] \subset (0, 1]$, $\zeta_n(I) = \sum_{i/n \in I} \xi_{n,i}$ gives general array sums, $\zeta_n(I) = \sum_{i/n \in I} 1(\xi_i > u_n)$ and $\zeta_T(I) =$

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