THE DETERMINISTIC EVOLUTION OF GENERAL BRANCHING POPULATIONS

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Probability theory has a strength that extends beyond probabilistic results. The precise formulation of probabilistic models may lead to intuitive arguments that reach further than even sophisticated mathematical analysis of deterministic models. This is well known from the use of Brownian motion in exhibiting solutions of partial differential equations. Another illustration is provided by population dynamics. Branching processes focus on probabilistic problems, and rely on probabilistic methods. But the expected evolution of general branching populations is an interesting topic in its own right, that has much in common with structured deterministic population dynamics. Arguments based on Markov renewal theory demonstrate a remarkable strength as compared to traditional, differential equations based approaches in establishing exponential growth and the ensuing stabilization of population composition of expected populations. This is described in this article, aimed at a broad mathematical readership.

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1 From Galton and Watson to Markov Population Processes

Recall Galton's famous formulation, more than a century ago, of the population extinction problem: "A large nation, of whom we will only concern ourselves with the adult males, N in number, and who each bear separate surnames, colonize a district. Their law of population is such that, in each generation, a_0 per cent of the adult males have no male children who reach adult life; a_1 have one such male child, a_2 have two; and so on up to a_5 who have five. Find (1) what proportion of the surnames will have become extinct after r generations; ..."

Already this historical and pre-exact wording has much of the flavour typical of modern mathematical population dynamics: its starting point is a description of individual behaviour, in this case a probabilistic description of reproduction, and the properties asked for concern the population as a whole - in this case an extinction probability. The latter is typical. In the biologically - not mathematically! - simple Galton-Watson processes that were born

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