ESTIMATION OF ANALYTIC FUNCTIONS

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In this paper we present a review of some results on nonparametric estimation of analytic functions and in particular derive minimax bounds under different conditions on these functions.

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1 Introduction

The aim of this paper is to present a review of some results about nonparametric estimation of analytic functions. A part of it is written in exposition style and summarizes some recent work of the author on the subject (detailed versions have already been published). The rest of the paper contains new results in the area. Sometimes the proofs are only outlined and will be published elsewhere.

Generally the problem looks as follows. We are given a class $\mathbf{F}$ of functions defined on a region $D \subset R^d$ and analytic in a vicinity of D. It means that all $f \in \mathbf{F}$ admit analytic continuation into a domain $G \supset D$ of the complex space C^d. To esimate an unknown function $f \in \mathbf{F}$ one makes observations X_ε. Consider as risk functions of estimators $\hat{f}$ for f the averaged $L_p(D)$-norms

$$\mathbf{E}_f\|\hat{f} - f\|_p = \mathbf{E}_f \left(\int_D |\hat{f}(t) - f(t)|dt\right)^{1/p}, \qquad 1 \le p < \infty,$$
$$\mathbf{E}_f\|\hat{f} - f\|_\infty = \mathbf{E}\{\sup_{x \in D} |\hat{f}(x) - f(x)|\},$$

where in the case of noncontinuous $\hat{f}$ the supremum is understood as an essential supremum. Put

$$\Delta_p(\varepsilon, \mathbf{F}) = \Delta(\mathbf{F}) = \inf_{\hat{f}} \sup_{f \in \mathbf{F}} \mathbf{E}_f\|\hat{f} - f\|_p.$$

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