CONFORMAL INVARIANCE, DROPLETS, AND ENTANGLEMENT

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Very brief surveys are presented of three topics of importance for interacting random systems, namely conformal invariance, droplets, and entanglement. For ease of description, the emphasis throughout is upon progress and open problems for the percolation model, rather than for the more general random-cluster model. Substantial recent progress has been made on each of these topics, as summarised here. Detailed bibliographies of recent work are included.

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1 Introduction

Rather than attempt to summarise the 'state of the art' in percolation and disordered systems, a task for many volumes, we concentrate in this short article on three areas of recent progress, namely conformal invariance, droplets, and entanglement. In each case, the target is to stimulate via a brief survey, rather than to present the details.

Much of the contents of this article may be expressed in terms of the random-cluster model, but for simplicity we consider here only the special case of percolation, defined as follows. Let $\mathcal{L}$ be a lattice in $\mathbb{R}^d$; that is, $\mathcal{L}$ is an infinite, connected, locally finite graph embedded in $\mathbb{R}^d$ which is invariant under translation by any basic unit vector. We write $\mathcal{L} = (\mathcal{V}, \mathcal{E})$, and we choose a vertex of $\mathcal{L}$ which we call the *origin*, denoted 0. The *cubic lattice*, denoted $\mathbb{Z}^d$, is the lattice in $\mathbb{R}^d$ with integer vertices and with edges joining pairs of vertices which are Euclidean distance 1 apart.

Let $0 \le p \le 1$. In bond percolation on $\mathcal{L}$, each edge is designated *open* with probability p, and *closed* otherwise, different edges receiving independent designations. In site percolation, it is the vertices of $\mathcal{L}$ rather than its edges which are designated open or closed. In either case, for $A, B \subseteq V$, we write $A \leftrightarrow B$ if there exists an open path joining some $a \in A$ to some $b \in B$,

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