ASYMPTOTICS IN QUANTUM STATISTICS

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Observations or measurements taken of a quantum system (a small number of fundamental particles) are inherently random. If the state of the system depends on unknown parameters, then the distribution of the outcome depends on these parameters too, and statistical inference problems result. Often one has a choice of what measurement to take, corresponding to different experimental set-ups or settings of measurement apparatus. This leads to a design problem—which measurement is best for a given statistical problem. This paper gives an introduction to this field in the most simple of settings, that of estimating the state of a spin-half particle given n independent copies of the particle. We show how in some cases asymptotically optimal measurements can be constructed. Other cases present interesting open problems, connected to the fact that for some models, quantum Fisher information is in some sense non-additive. In physical terms, we have non-locality without entanglement.

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1 Introduction

The fields of quantum statistics and quantum probability have a reputation for being esoteric. However, in our opinion, quantum mechanics, from which they surely derive, is a fascinating source of probabilistic and statistical models, unjustly little known to 'ordinary' statisticians and probabilists.

Quantum mechanics has two main ingredients: one deterministic, one random. In isolation from the outside world a quantum system evolves deterministically according to Schrödinger's equation. That is to say, it is described by a state or wave-function whose time evolution is the (reversible) solution of a differential equation. On the other hand, when this system comes into interaction with the outside world, as when for instance measurements are made of it (photons are counted by a photo-detector, tracks of particles observed in a cloud chamber, etc.) something random and irreversible takes place. The state of the system makes a random jump and the outside world contains a record of the jump. From the state of the system at the time of the interaction one can read off, according to certain rules, the probability distribution of the macroscopic outcomes and the new state of the system. See Penrose (1994) for an eloquent discussion of why there is something paradoxical in the peaceful coexistence of these two principles;