MARKOV CHAIN CONDITIONS FOR ADMISSIBILITY IN ESTIMATION PROBLEMS WITH QUADRATIC LOSS

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Consider the problem of estimating a parametric function when the loss is quadratic. Given an improper prior distribution, there is a formal Bayes estimator for the parametric function. Associated with the estimation problem and the improper prior is a symmetric Markov chain. It is shown that if the Markov chain is recurrent, then the formal Bayes estimator is admissible. This result is used to provide a new proof of the admissibility of Pitman's estimator of a location parameter in one and two dimensions.

AMS subject classifications: K1232, H3789.
Keywords and phrases: estimation, quadratic loss, admissibility, Markov chain, recurrence.

1 Introduction

In this paper we consider a classical parametric estimation problem when the loss is quadratic. Here attention is restricted to the so-called formal Bayes estimators – that is, estimators obtained as minimizers of the posterior risk calculated via a formal posterior distribution. Because the loss is quadratic, admissibility questions regarding such estimators are typically attacked using the explicit representation of the estimator as the posterior mean of the function to be estimated. Examples can be found in KARLIN (1958), STEIN (1959), ZIDEK (1970), PORTNOY (1971), BERGER and SRINIVASAN (1978), BROWN and HWANG (1982), EATON (1992), and HOBERT and ROBERT (1999).

To describe the problem of interest here, let $P(dx|\theta)$ be a statistical model on a sample space $\mathcal{X}$ where the parameter $\theta \in \Theta$ is unknown. That is, for each θ, $P(\cdot|\theta)$ is a probability measure on the Borel sets of $\mathcal{X}$. Both $\mathcal{X}$ and Θ are assumed to be Polish spaces with the natural σ-algebra. Given a real valued function $\phi(\theta)$ that is to be estimated, consider the loss function

$$L(a,\theta) = (a - \phi(\theta))^2, \qquad a \in R^1. \tag{1.1}$$

[1]Research was supported in part by National Science Foundation Grant DMS 96-26601. Research was supported in part by grants from CWI and NWO in the Netherlands.