STATISTICAL PROBLEMS INVOLVING PERMUTATIONS WITH RESTRICTED POSITIONS

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The rich world of permutation tests can be supplemented by a variety of applications where only some permutations are permitted. We consider two examples: testing independence with truncated data and testing extra-sensory perception with feedback. We review relevant literature on permanents, rook polynomials and complexity. The statistical applications call for new limit theorems. We prove a few of these and offer an approach to the rest via Stein's method. Tools from the proof of van der Waerden's permanent conjecture are applied to prove a natural monotonicity conjecture.

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1 Introduction

Definitive work on permutation testing by Willem van Zwet, his students and collaborators, has given us a rich collection of tools for probability and statistics. We have come upon a series of variations where randomization naturally takes place over a subset of all permutations. The present paper gives two examples of sets of permutations defined by restricting positions.

Throughout, a permutation π is represented in two-line notation

$$\begin{pmatrix} 1 & 2 & 3 & \cdots & n \\ \pi(1) & \pi(2) & \pi(3) & \cdots & \pi(n) \end{pmatrix}$$

with $\pi(i)$ referred to as the label at position i. The restrictions are specified by a zero-one matrix A_{ij} of dimension n with A_{ij} equal to one if and only if label j is permitted in position i. Let S_A be the set of all permitted permutations. Succinctly put:

$$S_A = \{\pi : \Pi_{i=1}^{n} A_{i\pi(i)} = 1\} \tag{1.1}$$

Thus if A is a matrix of all ones, S_A consists of all $n!$ permutations. Setting the diagonal of this A equal to zero results in derangement, permutations with no fixed points, i.e., no points i such that $\pi(i) = i$.