SOME REMARKS ON LIKELIHOOD FACTORIZATION

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Various types of likelihood factorization are reviewed and some general statistical consequences noted. In one broad class there is noniterative asymptotically efficient combination of information across factors via generalized least squares. This is used to discuss missing information in simple binary problems. It is shown that with observations on a 2 × 2 table supplemented by independent observations on each margin the maximum likelihood estimate of the odds ratio differs from that based on the complete table but is not asymptotically more efficient.

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1 Introduction

The likelihood function plays a key role in all approaches to the formal theory of parametric statistical inference and has implications also for semiparametric problems. We deal here primarily with the former. We consider a number of types of factorization of the likelihood function. For the important distinction between parameter-based and concentration-graph based factorizations and some applications to mixed binary and continuous variables, see Cox and Wermuth (1998).

We assume throughout what are known in some quarters as the British regularity conditions.

Suppose that for an observable random vector Y with observed value y there is a parametric statistical model leading to a likelihood function $L(\theta; y)$ with the parameter vector θ taking values in Ω_θ. Suppose further that we factorize the likelihood in the form

$$L(\theta; y) = L_1(\phi_1; y) L_2(\phi_2; y), \tag{1}$$

where (ϕ_1, ϕ_2) determines θ.

There is no essential loss of generality in restricting the discussion mostly to two factors. For sampling theory discussions we suppose that (1) is available for all y whereas for Bayesian calculations it is enough that (1) holds for the particular observed y.