QUADRATIC STATISTICS IN TESTING PROBLEMS OF LARGE DIMENSION

D.M. CHIBISOV[1]

Steklov Mathematical Institute

We consider testing a simple hypothesis about the mean vector of an N-variate normal distribution against shift alternatives in a Bayesian setting specifying a prior distribution of the mean vector under the alternative. We treat the problem asymptotically, as $N \to \infty$, and state fairly general conditions on the sequence of prior distributions under which the Bayes tests have asymptotically ellipsoidal acceptance regions.

AMS subject classifications: 62F03, 62F05, 62F15.
Keywords and phrases: Hypothesis testing, Bayes tests, multivariate normal distribution, large dimension.

1 Introduction

We consider testing a simple hypothesis about the mean vector of an N-variate normal distribution against shift alternatives in a Bayesian setup specified by a prior distribution of the mean vector under the alternative. Specifically, based on a single observation of an N-variate normal vector with identity covariance matrix we test the hypothesis that it has zero mean vector. We assume that for each N the prior distribution is the product of symmetric univariate distributions, or, in other words, under this prior the mean vector has independent symmetrically distributed components. Furthermore, we require these components to be in a certain sense asymptotically uniformly negligible. The result obtained can be viewed as an asymptotically complete class theorem saying that for this kind of alternatives in large dimension one can restrict oneself to tests with ellipsoidal acceptance regions. At the end of this section we give an example of a prior distribution for which our conditions fail.

The normal shift model of fixed dimension arises in asymptotic hypothesis testing problems about a multivariate parameter, the normal vector under consideration being the limit in distribution of a sequence of (vector-valued) asymptotically sufficient statistics, see, e.g., Roussas (1972), Chapter 6. (The general case of a known positive definite covariance matrix treated therein

---

[1] Research supported by the INTAS–RFBR grant 95–0099, by DFG-RFBR grant 98-01-04108, and by the RFBR Grant 96-15-96033 for Leading Scientific Scools..