LOCALIZATION AND DECAY OF CORRELATIONS FOR A PINNED LATTICE FREE FIELD IN DIMENSION TWO

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We prove that the two-dimensional harmonic crystal with a weak local pinning to a wall has finite variance and exponentially decaying correlations, regardless how weak the pinning is. The proof is based on an improved pressure estimate and an application of reflection positivity.

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1 A survey on models and questions

There is a natural class of generalizations of the standard random walks to higher dimensional "time", which are mainly considered in mathematical physics. To motivate them, we consider first a standard real valued random walk $X_0 = 0, X_1, \dots, X_n$. For simplicity, we assume that the distribution of the increments has a symmetric density f which is bounded and positive everywhere. Therefore, we can write $f(x) = \exp(-\phi(x))$, where ϕ is bounded from below, and symmetric. The joint density of $(X_1, \dots, X_n)$ is then $(x_1, \dots, x_n) \to \exp(-\sum_{i=1}^{n} \phi(x_i - x_{i-1}))$ where we put $x_0 = 0$. For the higher dimensional versions introduced below, it is usually more natural to look at random walks which are tied down at the endpoint, i.e. conditioned on $X_{n+1} = 0$. As we have assumed the increments to have a density, there is no problem to define that properly: The conditioned random walk has just the n-dimensional density

$$\frac{1}{Z_n} \exp\left[-\sum_{i=1}^{n+1} \phi(x_i - x_{i-1})\right] \tag{1}$$

where we now set $x_0 = x_{n+1} = 0$, and where Z_n is the appropriate norming

$$Z_n = \int \cdots \int \exp\left[-\sum_{i=1}^{n+1} \phi(x_i - x_{i-1})\right] dx_1 \dots dx_n$$

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