

# LIKELIHOODS AND PSEUDOLIKELIHOODS FOR MARKOV SPATIAL PROCESSES

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We study spatial random processes (mainly point processes in  $\mathbb{R}^d$ ) which are defined to satisfy various spatial analogues of the Markov conditional independence property. We explore some issues in statistical inference for such models, including likelihood and pseudolikelihood methods, and identifiability.

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## 1 Introduction

Markov point processes [75, 76] are a rich class of stochastic models for spatial patterns, with the virtue of being relatively tractable. They are defined to satisfy one of several spatial counterparts of the Markov conditional independence property. The likelihood takes a simple explicit form, apart from a difficult normalising factor. Indeed typically the likelihood is an exponential family, and the canonical sufficient statistic is often closely related to nonparametric spatial statistics. Typically each process is the equilibrium measure of an associated space-time Markov process; thus it is amenable to Markov Chain Monte Carlo simulation and bootstrap inference. Accordingly there is much current interest in exploring the potential applications of Markov point processes, which include spatial statistics, digital image analysis, and geostatistics.

The first half of this article is a condensed introduction to Markov point processes. The second half describes recent work by the author and collaborators (N.A. Cressie, N.I. Fisher, J. Møller, G. Nair, A. Särkkä and T.R. Turner) on finding new Markov models for different types of patterns, elaborating properties of these models, and performing statistical inference for spatial datasets using bootstrap, likelihood or pseudolikelihood methods.

## 2 Background

This section covers basic background about point process densities, Gibbs and Markov point processes, and conditional intensities.