

# FROM A TO Z: ASYMPTOTIC EXPANSIONS BY VAN ZWET

W. ALBERS

*University of Twente*

Refinements of first order asymptotic results are reviewed, with a number of Ph.D. projects supervised by van Zwet serving as stepping stones. Berry-Esseen bounds and Edgeworth expansions are discussed for  $R$ -,  $L$ - and  $U$ -statistics. After these special classes, the question about a general second order theory for asymptotically normal statistics is addressed. As a final topic, empirical Edgeworth expansions are considered.

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## 1 Introduction

In this paper an attempt is made to sketch van Zwet's contributions to the area of asymptotic expansions. Such a task is not particularly simple, as it concerns an expanding area in more than one sense, which also covers an impressively long period: from the early Seventies till now. (Hence the attempt to capture this comprehensive aspect in a literal manner in the title!) As a consequence, the resulting picture could easily become so loaded with details that the reader will have difficulty to focus, and the remaining impression will be blurred.

To avoid this from happening, we shall impose severe restrictions. In the first place, technical details will be dealt with rather loosely, and references will be given only sparingly. Both are amply available in the papers which we do refer to. Moreover, striving for completeness as far as references are concerned, would simply exhaust the available space and thus replace the intended sketch. A more essential restriction, however, is the fact that we shall not try to cover the whole area, but instead will select a single path through it. Our selection criterion, which seems suitable for an occasion like this, will be van Zwet's joint work on asymptotic expansions with quite a few of his students, during and following their Ph.D. projects under his guidance. Other contributions he made will typically only be included if these provided essential tools in these Ph.D. projects, or answered questions arising from such work.