EFFICIENT COUPLING ON THE CIRCLE

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Abstract

We consider efficient coupling specifically treating reversible Markov chains on the circle. We also show that "most" Markov chains with an efficient coupling have an "asymptotic monotone function".

Introduction

This paper is prompted by a recent paper of [BK] concerning efficient couplings of irreducible reversible continuous time Markov chains on a finite state space S. (We note those authors also studied efficiency questions for reflecting Brownian motion.) The starting point for this paper is the eigenvector expansion for the density function $p_t(x,y) = P^x(X_t = y)$ where $(X_t)_{t\geq 0}$ is our reversible Markov chain

$$p_t(x,y) = \pi(y) \sum_i e^{-\lambda_i t} \phi_i(x)\phi_i(y) \, .$$

Here π is the (unique) invariant distribution and ϕ_i are the right eigenvectors satisfying

$$Q\phi_i = -\lambda_i \phi_i \qquad i = 1 \cdots n$$
$$\sum \phi_i(x)\pi(x)\phi_j(x) = \delta_{ij},$$

where Q is the Q-matrix or generator of X. We refer to [AF] for important background materials on reversible Markov chains. Of course if the ϕ_i are ordered according to increasing λ_i, then $\lambda_1 = 0$, $\phi_1 \equiv 1$, and we have that

$$\sum_y |p_t(x,y) - \pi(y)| = O(e^{-\lambda_2 t})$$

and $\sum_y |p_t(x,y) - \pi(y)| = o(e^{-\lambda_2 t})$ if and only if $\phi_i(x) = 0$ for each eigenvector corresponding to λ_2.