Minimal Expected Ranks for the Secretary Problems With Uncertain Selection

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Abstract

A secretary problem which allows the applicant to refuse an offer of acceptance with probability $1-p$ $(0 < p < 1)$ is considered with the objective of minimizing the expected rank of the applicant selected. The optimal rule is derived and an explicit solution to the problem, as the number of applicants becomes infinite, is obtained. The memory-length-one rules of the problem are also discussed.

keywords. optimal stopping, relative ranks, memory-length-one rule.

1. Introduction and summary

Before discussing our problems, we state the basic framework of the secretary problem and review briefly the two standard problems. A set of n rankable applicants (1 being the best and n the worst) appear before us one at a time in random order with all $n!$ permutations equally likely. When the i-th applicant appears, we must decide either to accept (select) or reject it based on the observed rank of the applicant relative to those preceding it,$1 \leq i < n$. If no selection has been made prior to the n-th applicant, then the last one must be selected. An offer of selection is accepted with certainty by the applicant.

According to the criterion of optimality, the secretary problem is often distinguished into two standard problems ; the *rank minimization problem*, in which the objective is to minimize the expected (absolute) rank of the applicant selected and the *probability maximization problem*, in which the objective is to maximize the probability of selecting the best applicant. Chow et al.(1964) showed that, as $n \to \infty$, the minimal expected rank for the rank minimization problem tends to the value $\prod_{j=1}^{\infty}(1+\frac{2}{j})^{1/1+j} \approx 3.8695$. Bruss and Ferguson(1993) also considered this problem in the full information setting where the decision is based on the actual values associated with the applicants, assumed to be independent and identically distributed from a known distribution (see also Assaf and Samuel-Cahn(1996)). As for the probability maximization problem, Lindley(1961) showed that the limiting