Games against a prophet for stochastic processes

A. Meyerthole[a] and N. Schmitz[b*]

[a]Gesellschaft fr aktuarielle Beratung, Berrenrather Str. 197; 50937 Kln, Germany
[b]Institut fr Mathematische Statistik, University of Mnster, Einsteinstr. 62; D-48149 Mnster, Germany

Abstract

Two players (the „prophet" and the gambler) observe a uniformly bounded stochastic process $(X_s)_{s\in S}$. The prophet's maximal expected gain $E(\sup_{s\in S} X_s)$ is compared with the maximal expected gain $\sup_\tau EX_\tau$ of the gambler who is restricted to use stopping rules τ. Games against a prophet are two-person zero-sum games where the prophet picks the distribution and the gambler chooses a stopping rule. To obtain minimax-theorems for these games one has to admit mixed or randomized stopping rules. It is shown that mixed threshold stopping rules can be used to construct saddle-points for several cases.

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1. Introduction

Prophet theory is concerned with problems of the following kind: Two players, the *prophet* and the gambler, observe a (uniformly bounded) stochastic process $X_S = (X_s)_{s\in S}$ where $S = \{1, \dots, n\}$ (*finite horizon*), $S = \mathbb{N}$ (*infinite horizon*) and $S = [a, b] \subset [0, \infty)$ (*continuous time*) are the most interesting special cases. The gambler may stop this process at any time $s \in S$. His decision, leading to the reward X_s, may take into account the previous observations $X_t, t \leq s$, but not the future ones, i.e. he is restricted to use non-anticipating stopping functions τ. The supremum over the expected

*Corresponding author; E-mail: SCHMNOR@MATH.UNI-MUENSTER.DE