ON THE ESTIMATION OF SURVIVAL FUNCTIONS UNDER A STOCHASTIC ORDER CONSTRAINT

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ABSTRACT. Consider distribution functions F and G and suppose that $F(x) \leq G(x)$ for all x. The problem of estimating F or G, or both, arises quite naturally in applications. For example, in corrosion engineering it is of interest to estimate the pitting times of metals under two different strengths of corrosive environments. The empirical distribution functions F_m and G_n will not necessary satisfy the order constraint imposed by the experimental conditions. Lo (1987) proposed the estimators $\hat{F}_m = \min(F_m, G_n)$ and $\hat{G}_n = \max(F_m, G_n)$, which satisfy the constraint of interest, and showed that these estimator are asymptotically minimax, under suitable conditions, for a large class of loss functions. Although $\hat{F}_m$ and $\hat{G}_n$ are strongly uniformly consistent when both m and n tend to infinity, neither one is when only m or n go to infinity. Here, the estimators F_m^* (G_n^*) are proposed which are strongly uniformly consistent for F (G) when only m $(n) \to \infty$. The case of censored data is also considered. Under suitable conditions, weak convergence of the processes $\{\sqrt{m}(F_m^*(x) - F(x)), \quad 0 < x < \infty, \quad m = 1, \ldots\}$ and $\{\sqrt{n}(G_n^*(x) - G(x)), \quad 0 < x < \infty, \quad n = 1, \ldots\}$ is demonstrated. As a consequence, asymptotic confidence bands are obtained. For testing the hypothesis of identical distributions against a stochastic order alternative, the asymptotic distribution of the estimators under the assumption that $F(x) = G(x)$ for all x is also discussed. The results of a Monte-Carlo study show that the new estimators perform better than $\hat{F}_m$ and $\hat{G}_n$ and the non-parametric maximum likelihood estimators in terms of bias and mean squared error for a large class of examples.

1. INTRODUCTION

In many experimental sciences it is often of interest to estimate lifetime of experimental units when two different treatments are applied. For example, in corrosion engineering, the times until pitting of metals immersed in a corrosive environment are measured under two different solution corrosivities. Shibata and Takeyama (1977), for example, present data which strongly supports the belief that the times until pitting should be shorter in some sense, for the more corrosive environment. In toxicity studies, cells are