OPTIMALITY RESULTS IN MULTIPLE HYPOTHESIS TESTING

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ABSTRACT. Multiple hypothesis testing occurs in a vast variety of fields and for a vast variety of purposes. Optimality results are relatively sparse in this area compared to results for tests of individual hypotheses. This paper restricts consideration to cases in which a finite number of parameters are involved, in which conclusions are desired for each parameter separately, and in which directional inference may or may not be involved. The paper does not deal with optimal design, with tests of composite hypotheses without further resolution, nor with sequential analysis or ranking and selection procedures. It is primarily a historical survey, concentrating on work of Erich Lehmann, but relating his optimality results to more recent developments, primarily in stepwise testing.

1. INTRODUCTION

Testing of more than one hypothesis simultaneously is widely practiced in many fields and for many purposes. While theory and methods in this area originally arose in connection with relatively small numbers of treatments being evaluated or compared, more recently there has been a great increase in applications to situations in which massive numbers of hypotheses are being considered jointly, such as in large surveys (for example, the National Assessment of Educational Progress–see Beaton and Zwick, 1992), signal compression, microarray analysis, and astronomy. In some of these situations, the number of hypotheses ranges into the tens of thousands and higher.

An alternative way of looking at multiple hypothesis testing is as a multiple decision problem. It turns out to be useful to consider such problems from both multiple testing and multiple decision perspectives, and sometimes in fact to combine these two perspectives. These dual perspectives and combinations will be illustrated.