LECTURE 9

Curve Estimation and Long-Range Dependence

It is of some interest to see what one can say about probability density estimates in some domain of long-range dependence. Let us consider a stationary process Y_k, with a density function, given by

$$Y_k = G(X_k)$$

with X_k Gaussian and stationary. One can show [see Ibragimov and Rozanov (1978)] that if the correlation [of (X_k)]

$$r(s) \simeq q|s|^{-\alpha}, \qquad \alpha > 1, \tag{9.1}$$

then X_k is a process with asymptotic correlation zero and with corresponding mixing coefficient $\rho(s) = o(|s|^{-\beta})$ for $\beta < \alpha - 1$. This is also true of the process Y_k since it is obtained by an instantaneous function applied to the process X_k. Suppose we wish to consider a kernel estimate of the density function of Y_k,

$$f_n(y) = \frac{1}{nb(n)} \sum_{k=1}^{n} \omega\left(\frac{y - Y_k}{b(n)}\right).$$

A theorem of Bradley (1983) as applied here would give us the usual result on asymptotic distribution of the estimates if ω is nonnegative, bounded and band limited with integral one.

Let us now consider the case in which α in (9.1) is such that $0 < \alpha < 1$. The process X_k is then long-range dependent by the remarks in the previous section. If we expand in terms of Hermite polynomials as in the last section,

$$\omega\left(\frac{y - G(x)}{b(n)}\right) = \sum c_{j,n} H_j(x)$$

with

$$c_{j,n} = \frac{1}{j!} \int \omega\left(\frac{y - G(x)}{b(n)}\right) H_j(x) \frac{\exp(-x^2/2)}{\sqrt{2\pi}}\, dx.$$