CHAPTER 10

The New Likelihoods and the Neyman–Scott Problems

10.1. Introduction. The traditional method of using the likelihood to make inference about the parameter of interest is to use the so-called profile likelihood, which is the likelihood maximized with respect to the nuisance parameters. It has been known for a long time that this is a wrong thing to do if there are many nuisance parameters. The Neyman–Scott examples provide a dramatic example of this. In one of them, maximizing the profile likelihood, which is the same thing as using the mle, provides an inconsistent estimate of the parameter of interest.

Two modifications of profile likelihood have been proposed recently. Conditional likelihood, owing to Cox and Reid (1987) and adjusted likelihood, due to McCullagh and Tibshirani (1990), both try to modify the profile likelihood so that it may be expected to behave more like an honest likelihood. Both have been tried on the two Neyman–Scott examples we discuss here.

In Section 10.2 we introduce and briefly study these new likelihoods by methods of higher order asymptotics. In Section 10.3 we introduce two Neyman–Scott examples, as well as a general formulation, and introduce estimates which are FOE in a sense appropriate for these problems. We then apply the new likelihoods to these examples and note that they fail to provide FOE estimates. We suggest that they are not the right answers to these problems. A modified version of the general Neyman–Scott problem is posed for which higher order asymptotics seems to be the right tool and the two new likelihoods may do better than profile likelihood.

10.2. Conditional and adjusted likelihood. We consider $\theta = (\theta_1, \theta_2)$, where θ_1 is the parameter of interest and θ_2 is the nuisance parameter. We will require θ_1 and θ_2 to be orthogonal and so, as mentioned in Chapters 8 and 9, θ_1 will need to be real valued.