CHAPTER 8

Magic Formula, Bartlett Correction and Matching Probabilities

8.1. Introduction. The magic formula of Barndorff-Nielsen (1983) is a beautiful formula in higher order asymptotics for calculating the density of the mle, or its conditional density given an ancillary statistic, which was called a magic formula because it seemed magical when it first appeared. It still retains some of its magical quality, though it is much better understood now than when it first appeared. For example it is still unclear why it is so good an approximation even for very small values of n like $n = 3$ or 4. Barndorff-Nielsen and Cox (1989) provide a nice exposition as well as many interesting applications. The magic formula is a special case of saddle point formulas. For an excellent introduction as well as exposition of recent results, see Reid (1988) and Field and Ronchetti (1990). In Section 8.2 we present the magic formula in the general case, along with a proof in a simple case, and pose a number of open problems.

Bartlett's correction is another formula in higher order asymptotics with a magical quality. We provide a Bayesian argument in Section 8.3 which is natural, general and rigorous. The proof makes clear that the correction is natural in the Bayesian context and the frequentist correction can be derived from this. In Section 8.4 we use the Bayesian argument to generate confidence sets which have the correct coverage probability of $1 - \alpha$ up to $O(n^{-2})$ (uniformly on compact θ sets). These sets have the attractive property of also having posterior probability of $1 - \alpha$ to $O(n^{-2})$ of covering the true value of θ. We show also how the frequentist Bartlett correction can be calculated through a Bayesian route.

8.2. The magic formula. Consider i.i.d. continuous r.v.'s $X_1, X_2, \ldots, X_n$ with (linear) exponential density (with respect to Lebesgue measure)

$$p(x|\theta) = c(\theta)e^{\theta x}A(x).$$