CHAPTER 6

Third Order Efficiency, Admissibility and Minimaxity

6.1. Third order efficiency in the general case. We assume regularity conditions on $p(x)|\theta)$ so that Theorem 5.1d holds, and assume the following three conditions on estimates:

CONDITION 1. $E\{(T_n - \theta)^2|\theta\} = n^{-1}I^{-1}(\theta) + n^{-2}g(\theta) + o(n^{-2})$, uniformly in compact θ sets where $g(\theta)$ is a continuous function of θ.

CONDITION 2. $E\{(T_n - \theta)|\theta\} = n^{-1}b(\theta) + O(n^{-(1+\varepsilon)})$, uniformly in compact θ sets where $b(\theta)$ is continuously differentiable and $\varepsilon > 0$.

CONDITION 3. $\sup_{\theta \in [a,b]} E\{(T - \theta)^4|\theta\} \leq M_{a,b} < \infty$ bound intervals $[a, b]$.

We assume $\hat{\theta}$ satisfies these conditions also. That expectation of $\hat{\theta}$ and variance of $\hat{\theta}$ have expansions in powers of n^{-1} follows from the regularity conditions assumed earlier. The expansions agree with those obtained by the delta method. If b_0 and g_0 stand for b and g when T_n is replaced by $\hat{\theta}$, we do not need to make the additional assumption that b_0 is continuously differentiable and g_0 is continuous. The quantities b_0 and g_0 as calculated earlier for curved exponentials have the same expressions in the general case.

Fix θ_0 and introduce a sequence of priors $\pi_n \in D_\infty$, concentrating on the interval (a_n, b_n) with $a_n = \theta_0 - (\log n)^{1/4}$, $b_n + \theta_0 + (\log n)^{14}$. [In fact, choose the prior exhibited in (5.6).] It can be shown that Theorem 5.1d continues to hold if π is replaced by π_n. Write B_n', corresponding to π_n, in the form

(6.1) $$B_n' = \hat{\theta} + d_n(\tilde{\theta})/n.$$