CHAPTER 2

Edgeworth Expansions, Curved Exponentials and Fisher Consistent Estimates

2.1. Edgeworth expansions for functions of sample mean. To explain what the expansions for asymptotic mean and variance mean, we need the concept of Edgeworth expansions.

Let X_i, $i = 1, 2, \dots, n$, be m-dimensional, i.i.d., $f_1(\cdot), \dots, f_k(\cdot)$, k real valued (measurable) functions on R^m and

$$Z_j = \big(f_1(X_j), f_2(X_j), \dots, f_k(X_j)\big)$$

with finite mean $\mu = E(Z_j)$ and finite positive definite dispersion matrix $[\sigma_{jj'}]$. The ith coordinates of Z_j and μ are Z_{ij} and μ_i. Let

$$\bar{Z} = \frac{1}{n}\sum_1^n Z_j, \qquad T \equiv T_n = H(\bar{Z}),$$

where H is a real valued function which is $(s-1)$ times continuously differentiable in a neighborhood of μ, $s \geq 2$.

By Taylor expansion around μ,

$$\begin{aligned} H(\bar{Z}) = H(\mu) &+ \sum l_i(\bar{Z}_i - \mu_i) + \sum l_{ii'}(\bar{Z}_i - \mu_i)(\bar{Z}_{i'} - \mu_{i'}) \\ &+ \cdots + \sum l_{i_1, i_2, \dots, i_{s-1}}\big(\bar{Z}_{i_1} - \mu_{i_1}\big)\big(\bar{Z}_{i_2} - \mu_{i_2}\big)\cdots \\ &\qquad\qquad \times\big(\bar{Z}_{i_{s-1}} - \mu_{i_{s-1}}\big) + R_n \end{aligned} \tag{2.1a}$$

$$\underset{\text{def}}{=} H_1(\bar{Z}) + R_n,$$

where

$$l_i = \left.\frac{\partial H(Z)}{\partial z_i}\right|_{\mu}, \qquad l_{ii'} = \left.\frac{\partial^2 H}{\partial z_i\, \partial z_{i'}}\right|_{\mu}, \quad \text{and so on,}$$