CHAPTER 1

Introduction

1.1. Overview. Higher order asymptotics deals with two sorts of closely related things. First, there are questions of approximation. One is concerned with expansions or inequalities for a distribution function, an asymptotic variance, the posterior density and integrated Bayes risk. Second, there are inferential issues. These involve, among other things, the application of the first set of ideas of the study of higher order efficiency, admissibility and minimaxity, conditional and adjusted likelihood and construction of noninformative priors by approximately matching posterior and frequentist probability. In the matter of expansions, it is as important to have usable, explicit formulas as a rigorous proof that the expansions are valid in the sense of truly approximating a target quantity up to the claimed degree of accuracy.

Classical asymptotics is based on the notion of asymptotic distribution, often derived from the central limit theorem, and usually the approximations are correct up to $O(n^{-1/2})$, where n is the sample size. Higher order asymptotics provides refinements based on asymptotic expansions of the distribution or density function of an estimate and the posterior density function of the parameter. Posterior expansions are refinements of the Bernstein–von Mises theorem on asymptotic normality of the posterior, whereas the other expansions are rooted in the Edgeworth theory, which is itself a refinement of the central limit theorem.

When higher order asymptotics is correct up to $o(n^{-1/2})$, it is second order asymptotics. When further terms are picked up, so that the asymptotics is correct up to $o(n^{-1})$, it is third order asymptotics. In his pioneering papers, C. R. Rao coined the term second order efficiency for a concept that would now be called third order efficiency. The new terminology is essentially owing to Pfanzagl and Takeuchi.

The stress in the subsequent chapters is on basic concepts and main results, with enough technical details to make applications to specific examples fairly easy. For the main results, we provide a proof, or, where a proof is too long or technical, a sketch of the argument and a reference to where details are available. We do not strive for maximum generality.