Chapter 2

Estimation in the LMCD Assuming Normally Distributed Errors and Complete Data

In this chapter we consider maximum likelihood inference for the parameters of the Linear Model for Correlated Data (LMCD) of Section 1.2 under the additional assumption that the error vectors are normally distributed. To present the basic ideas, we will assume that the data are obtained from a complete study with each $n_i = n$ and we restrict attention to the setting in which Σ_i is assumed not to depend on covariates, so that $\Sigma_i = \Sigma$ is the same for all i. We will consider both maximum likelihood (ML) and restricted maximum likelihood (REML) estimation when Σ is unstructured. Inference under more restrictive parametric models for Σ is briefly discussed at the end of this chapter. Estimation with unbalanced designs and/or missing data will be taken up in Chapter 3.

2.1 ML Estimation of β and Σ

Suppose that in the LMCD we additionally assume that given X_i, Y_i, $i = 1, \ldots, N$, are N copies from a multivariate normal random vector. Then, under this additional assumption, the likelihood of (β, Σ) is given by

$$\mathcal{L}(\beta, \Sigma) = |\Sigma|^{-N/2} e^{-\sum_{i=1}^{N}(Y_i - X_i\beta)^T \Sigma^{-1}(Y_i - X_i\beta)/2}.$$