Chapter 8

Other estimation methods for GLMMs

8.1 Introduction

In this chapter I consider alternatives to maximum likelihood, which we saw are computationally demanding. Generalized estimating equations (GEEs) are a popular approach to fitting a related class of models. I also consider Penalized Quasi-Likelihood (PQL) and variants and the more general topic of estimating equations as a unifying theme. In a number of points in this chapter I fall back on the linear mixed model to give insight in a situation in which calculations are a bit more straightforward.

8.2 Generalized estimating equations

Generalized Estimating Equations (GEEs) is a computationally less demanding method than ML estimation. It has mainly been designed as a marginal modeling approach for longitudinal data, wherein data is collected on "subjects" on two or more occasions. It works best when the number of occasions is small compared to the number of subjects.

To motivate the ideas it is easiest to return to linear models. Consider the standard linear model with a full-rank $\mathbf{X}$ matrix:

$$\mathrm{E}[\mathbf{Y}] = \mathbf{X}\boldsymbol{\beta}$$

with well-known ordinary least squares estimator (OLSE)

$$\hat{\boldsymbol{\beta}}_{ols} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}. \tag{8.1}$$

The OLSE is optimal (best linear unbiased or best unbiased) in the situation in which the data can be assumed to be homoscedastic and uncorrelated (with a further normality assumption to get best unbiased). However, it works well in many situations for which those assumptions are not met. In particular it is always unbi-