Chapter 7

Maximum likelihood for GLMMs

7.1 Introduction

As noted in Chapter 1, creation of a GLMM by incorporating random factors in the linear predictor of a GLM leads to difficult-to-handle likelihoods. This is first laid out more carefully in a simple example and then general approaches to maximum likelihood are described.

7.2 A simple example

To fix ideas consider the following logit-normal example:

\[ Y_{ij} | u \sim \text{indep. Bernoulli}(p_{ij}), \quad i = 1, 2, \ldots, q; \quad j = 1, 2, \ldots, n, \]

\[ \logit(p_{ij}) = \beta x_{ij} + u_i, \]

\[ u_i \sim \text{indep. } \mathcal{N}(0, \sigma^2). \]

In this scenario there are \( q \) clusters, each with \( n \) observations, a logit link and a single random and single fixed factor. The random effects, \( u_i \), are assumed to be i.i.d. normally distributed.

The example is so simplified it is a stretch to come up with a realistic situation it might reflect, but here is an attempt. Suppose we record \( Y_{ij} = 1 \) if a subject’s blood pressure decreases on day \( j \) of treatment with a blood pressure drug at dose \( x_{ij} \), and is 0 otherwise. There are \( q \) individuals tested, each at \( n \) different doses. Since the intercept is zero, when the dose is 0 and for \( u_i \) equal to its mean of zero, the probability of a decrease is 0.5. The interpretation of \( u_i \) is the person-specific propensity to decrease or increase blood pressure in response to treatment (a type of individual-specific placebo effect).

Since the model is specified conditionally, it is easiest to derive the likelihood