# Chapter 2

# Linear Mixed Models (LMMs)

## 2.1 Introduction

There are a number of books that cover the details of linear mixed models, including McCulloch and Searle (2000); Searle et al. (1992); Verbeke and Molenberghs (2000) so I will not attempt to cover the topic in detail here. However, I do want to point out several facets of linear mixed models and their estimation that are relevant to generalized linear mixed models and establish some basic notation. Again, I begin with an example, this one quite simple.

## 2.2 Example: Propranolol and hypertension

Below (Table 2.1) are data from an early, double-blind trial of the effect of a drug, Propranolol, on hypertension. Blood pressure was measured after administration of the drug and a placebo both in the upright and recumbent positions. There are two main questions of interest. First, does Propranolol have the same influence in recumbent and upright positions (i.e, is there a lack of interaction) and second, if the answer to the first question is yes, is it effective?

If we let $Y_{ijk}$ denote the blood pressure measurement on the $k$th individual, $i$th position and $j$th drug condition, then the standard model for such an analysis is

$$\begin{aligned} Y_{ijk}|p_k &\sim \text{indep. } \mathcal{N}(\mu_{ijk}, \sigma^2), \\ \mu_{ijk} &= \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + p_k, \end{aligned} \tag{2.1}$$

where the vertical bar between $Y_{ijk}$ and $p_k$ indicates that the specification is conditional on the $p_k$. To this we add the important assumption that the person effects, $p_k$, follow a distribution:

$$p_k \sim \text{i.i.d. } \mathcal{N}(0, \sigma_p^2). \tag{2.2}$$

The mere declaration of the $p_k$ as random variables, as contrasted with treating them as fixed, unknown parameters, induces a correlation between measurements