

## CHAPTER 1

# The Wide Scope

The mixture model has long been a challenge to the statistician, whether beginner, practitioner or theoretician. Recent times have seen great advances in our understanding of the some basic mathematical features of this model, and these notes are meant to be a unification of the work I have carried out, jointly with many wonderful collaborators, in this area. Based on lectures given in 1993 at a regional conference of the Conference Board of the Mathematical Sciences, the notes are directed toward a mixed audience of advanced graduate students and research workers in this and related areas. For the sake of newcomers to the mixture model, I will attempt to be complete enough for the text to make sense in itself, but must at some points refer the reader to other more extensive treatments.

Unfortunately, the goal of timeliness in the end also forced some truncation of the subject matter in the original lecture notes. On the other hand, some subjects have been given enhanced development because they are truly new, and the audience I have in mind will appreciate a deeper presentation of background and of the beautiful geometric structures of the model.

The first chapter of these notes, corresponding to Lecture 1, lays out the mixture landscape as I see it, and the practical side of my motivation for interest in the area. There are two major points:

- There are *many* statistical topics, some quite extensive by themselves, that can rightly be called mixture model subtopics. They all share the mixture model structure, and have similar inferential goals. One of the themes of this chapter is, therefore, that the many names for the mixture model hide its universality. In these notes I am aiming for the universal aspects that lie beneath.
- There is a nonparametric approach to maximum likelihood in the mixture model that gives us an extremely powerful set of tools to use, both in a nonparametric approach and in diagnosing the ailments of parametric approaches. I might add that, to my mind, it is mathematically elegant and fun.