LECTURE 10

Open Questions

There are a number of open questions that arise naturally. In Lectures 8 and 9, results on long-range dependence were discussed. However, all these were discussed for processes subordinate to Gaussian stationary processes. It would be of some interest to obtain appropriate limit theorems in a domain broader than that of processes subordinate to Gaussian processes, particularly when dealing with a continuous time parameter. One should note that all the results discussed in these notes have been for a discrete time parameter.

In Lecture 3, results of a global character for probability density estimates were obtained making use of the result of Komlós, Major and Tusnády when dealing with independent observations. It is natural to ask whether one can get a sufficiently broad version of a strong invariance principle with an error term good enough so as to get comparable results in the case of short-range dependence.

Lecture 7 considered conditions for asymptotic normality of spectral density estimates. However, these conditions still seem rather far from what one might think are natural boundaries for the domain of validity of asymptotic normality for these estimates. Herrndorf (1983) gave an interesting example of a class of strongly mixing sequences (with exponentially decaying mixing coefficients) that do not satisfy the central limit theorem. One would like to know whether a similar phenomenon could occur for spectral density estimates (just as for partial sums in Herrndorf's example).

The extent to which corresponding results hold for random fields as well as sequences is a natural question. It is clear that one has to formulate questions in terms of an appropriate version of a strong mixing condition when dealing with dependence, since otherwise results will possibly have a trivial character.

In Lecture 2, questions relating to boundary behavior were mentioned. Such questions are of special interest when dealing with the asymptotic behavior of smoothing splines of any fixed order as the basis for regression estimates. In this respect, it would be worthwhile elaborating the methods used in Messer and Goldstein (1989). These questions are more complicated and interesting in