CHAPTER 9

Noninformative Priors

9.1. Introduction. Determination of noninformative priors obtained by matching posterior and frequentist probabilities depends on higher order asymptotics. The noninformative priors introduced by Bernardo (1979) and Berger and Bernardo (1989) are based on first order asymptotics, namely, the asymptotic normality of posteriors. We discuss both briefly in this chapter, partly for the sake of completeness and partly because for a small number of parameters, the choice of a prior is usually important only for a moderately large n, $n = 10$ or so; for large n, the asymptotic normality of posterior (under regularity conditions on p and smoothness of π) leads to practically no influence of the prior on the posterior. So the choice of such priors does seem to be within the domain of higher order asymptotics.

Bernardo (1979) and Berger and Bernardo (1989) have called their noninformative priors reference priors. This seems an appropriate terminology for all noninformative priors. They may be viewed as an origin or a reference point against which a given prior, incorporating subjective opinion, can be judged. Moreover the posterior based on such a prior provides a Bayesian reporting of data which is close to being "objective" or free from prior subjective belief as far as this is permitted in the Bayesian paradigm. For both these reasons they are likely to play an important role in Bayesian analysis. [See also Berger (1985).] An emerging terminology for them is default or automatic priors, which indicates their use in quick Bayesian data analysis in default of a fully subjective Bayesian treatment.

As discussed in Ghosh and Mukerjee (1992b), there are usually four notions associated with noninformative priors:

1. Maximizing entropy or minimizing information.
2. Matching what a frequentist might do (since, one may argue, that is how a noninformative Bayesian should act).
3. Invariance.
4. Minimaxity (in a weak form).

In the next two sections we will be discussing notions 1 and 2. Asymptotically, consideration of invariance or minimaxity leads one to the Jeffreys prior, which will also appear as an entropy minimizing prior in the next