LECTURE 6

Probability Density and Regression Estimation in the Case of Short-Range Dependence

A result of Bradley (1986) adapted for kernel density estimates assumes that (X_k) is a strictly stationary sequence with absolutely continuous marginal distribution. The marginal density is assumed continuous and positive. The joint probability distribution of (X_0, X_k) is also assumed absolutely continuous with a continuous joint density. The weight function of a kernel estimate ω is assumed to be nonnegative Borel and to satisfy (i) $\int \omega(u)\,du = 1$; (ii) $\int \omega^{2+\delta}(u)\,du < \infty$ for some $\delta > 0$; (iii) ω has bounded support. If the process (X_k) has asymptotic correlation zero property and $\sum \rho(2^n) < \infty$, where $\rho(j)$ is the correlation of X_0 and X_j, the probability density function estimates

$$f_n(x_s) = (nb_n)^{-1} \sum_{k=1}^{n} \omega\left(\frac{x_s - X_k}{b_n} \right),$$

$s = 1, 2, \ldots, L$, are such that $(nb_n)^{1/2}(f_n(x_s) - Ef_n(x_s))$ if $n \to \infty$, $b_n \to 0$, $nb_n \to \infty$ are asymptotically jointly normal and independent with means zero and variances

$$f(x_s) \int \omega^2(u)\,du.$$

The argument for this result is based on the extension of a central limit theorem for stationary sequences with asymptotic correlation zero condition to triangular arrays.

Here we will see to what extent results of a local character for probability density and regression estimates hold just as in the independent case when one