LECTURE 5

Measures of Short-Range Dependence

There are a number of ways of setting up conditions under which versions of the classical limit theorems still hold for dependent sequences. We shall be mainly concerned with the central theorem for partial sums of the random variables of the sequence. One of these ways is to consider some type of mixing condition. With this in mind we consider the following measures of dependence for two sub-σ-fields $\mathscr{A} \subset \mathscr{F}$, $\mathscr{B} \subset \mathscr{F}$ of a probability space $(\Omega, \mathscr{F}, P)$. They are

$$\alpha(\mathscr{A}, \mathscr{B}) = \sup |P(A \cap B) - P(A)P(B)|, \qquad A \in \mathscr{A},\ B \in \mathscr{B},$$

$$\phi(\mathscr{A}, \mathscr{B}) = \sup |P(B|A) - P(B)|, \qquad A \in \mathscr{A},\ B \in \mathscr{B},\ P(A) > 0,$$

$$\phi_{\text{rev}}(\mathscr{A}, \mathscr{B}) = \phi(\mathscr{B}, \mathscr{A}),$$

$$\psi(A, B) = \sup \frac{|P(A \cap B) - P(A)P(B)|}{P(A)P(B)}, \qquad A \in \mathscr{A},\ B \in \mathscr{B},$$

$$\rho(\mathscr{A}, \mathscr{B}) = \sup |\text{corr}(X, Y)|, \qquad X \in L_2(\mathscr{A}),\ Y \in L_2(\mathscr{B}),\ X, Y \text{ real},$$

$$\beta(\mathscr{A}, \mathscr{B}) = \sup \frac{1}{2} \sum_{i-1}^{I} \sum_{j-1}^{J} |P(A_i \cap B_j) - P(A_i)P(B_j)|,$$

with the supremum taken over partitions $\{A_i, i \in I\}$ and $\{B_j, j \in J\}$ each of Ω but with $A_i \in \mathscr{A}$, $B_j \in \mathscr{B}$. Here $L_2(\mathscr{A})$ denotes the set of square integrable functions measurable with respect to the σ-field $\mathscr{A}$.

Let us now consider a stationary sequence of random variables X_k, $k = \dots, -1, 0, 1, \dots$. $\mathscr{F}$ is the σ-field generated by the random variables of the process $\mathscr{B}(X_j, -\infty < j < \infty)$. Set

$$\mathscr{B}_n = \mathscr{B}(X_j, j \le n),$$

$$\mathscr{F}_m = \mathscr{B}(X_j, j \ge m).$$