LECTURE 3

Global Measures of Deviation

There is by now a large literature on lower bounds attainable in nonparametric density or regression estimation. Discussion of these questions can be found in the works of Farrell (1972), Stone (1980, 1982) and Hall (1989). We shall follow an exposition given by Hall (1989) when one estimates an unknown scalar. Consider models f from a family $\mathscr{F}$. L_f will denote the likelihood under model f. Given $f_0, f_1 \in \mathscr{F}$ let

$$d = \begin{cases} 0 & \text{if } L_{f_0}/L_{f_1} \geq 1, \\ 1 & \text{otherwise.} \end{cases}$$

Further let

$$c = \tfrac{1}{2} \inf_{n \geq n_0} \left\{ P_{f_0}(d = 1) + P_{f_1}(d = 0) \right\} \tag{3.1}$$

or

$$c = \left\{ 4 \sup_{n \geq n_0} E_{f_0}\left(L_{f_1}/L_{f_0} \right)^2 \right\}^{-1}. \tag{3.2}$$

The interest is in an estimate $\hat{\theta}$ of the unknown scalar $\theta = \theta(f)$. Set $b_n = \frac{1}{2}|\theta(f_0) - \theta(f_1)|$. Θ is the set of all nonparametric estimators of θ.

Two models f_0 and f_1 are selected from $\mathscr{F}$. It is usually the case that f_0 is fixed and f_1 converges to f_0 at an appropriate rate. A lower bound for the convergence rate of $\hat{\theta}$ to θ is given by b_n in the sense given by (3.3). In effect the basic issue centers on the ability to discriminate between f_1 and f_0.

THEOREM. *One can show that*

$$\inf_{\hat{\theta} \in \Theta} \sup_{f \in \mathscr{F}} P_f\left(|\hat{\theta} - \theta| \geq b_n \right) \geq c \tag{3.3}$$

for all $n \geq n_0$.

It should be understood that we are interested in what happens as $n \to \infty$. The theorem has a minimax character typical of many results in this direction.