

## LECTURE 2

# Local Asymptotics

Historically, the greatest attention in the statistical literature has been paid to parametric problems. This is especially so in the case of independent identically distributed observations. In most of this development, a very detailed assumption is made about the common distribution of the observations, for example, a Gaussian distribution. The typical result is that under appropriate smoothness assumptions on the common distribution, reasonable estimates of the parameters converge at the rate  $n^{-1/2}$ . There are, of course, nonparametric problems of some vintage, for example, that are concerned with estimation of the common distribution function of the observations. In that context one still has convergence at the rate  $n^{-1/2}$ . In estimating the distribution function, one is estimating what one might call a global function. In recent years, there has been considerable interest in function estimation with a local character, for example, of a probability density, a regression function or a spectral density. There, typically the rates of convergence are slower. There is by now an enormous literature and we shall by no means try to cover it. Unfortunately we may not be fully accurate in attributing ideas or developments to those most responsible. Perhaps the best we can do is to follow a few suggestive ideas that touch on many developments and give some insight into typical results and directions. It should also be noted that when one tries to extend results for such local curve estimates to dependent observations with short-range dependence, many of the asymptotic results have the same character as in the case of independent observations. A basic motivation for investigations of this type is a skepticism or doubt relative to the usual assumptions in the classical finite parameter theory. The usual assumption of a specific finite parameter family of densities is regarded as unconvincing. The idea is that the data should be used to estimate or test the distributional or regression form. Practically all results have an asymptotic character and before applying such results one should try to get an idea of the extent to which such asymptotics provide useful finite sample approximations. We cannot pursue this difficult but important question here. Our exposition will mainly center on