LECTURE 1

Origins

In a set of lectures on curve estimates in the case of independent and dependent observations, it is perhaps appropriate to have initial comments that are in part reminiscences of earlier days and work in the area before proceeding to a discussion of current problems and research. This can provide a motivation for the later development. It is rather doubtful whether comments of this sort can be taken seriously as scientific history. Perhaps attempts at such reconstruction can only convince one of the difficulties involved in writing history. Nonetheless they can give a personal perspective of the time.

During and after World War II there was a good deal of interest in dependent processes as models. One of the models examined probabilistically even before the war was that of a weakly stationary sequence of random variables x_t, $t = \dots, -1, 0, 1, \dots,$ that is, a sequence with constant mean and covariance function

$$r_{n-m} = \operatorname{cov}(x_n, x_m)$$

depending only on the time difference $n - m$. Such a sequence r_m is positive definite so that for any complex constants c_j,

$$\sum_{j,k=1}^{n} c_j r_{j-k} \bar{c}_k \geq 0$$

for each positive integer n. An old result due to Herglotz (1911) states that the covariances are Fourier–Stieltjes coefficients of a bounded nondecreasing function G,

$$r_n = \int_{-\pi}^{\pi} e^{in\lambda} \, dG(\lambda).$$

In current terminology, $G(\lambda)$ is called the spectral distribution function of the random sequence x_n. There is a continuous time analogue of this result in