SECTION 11

Least Absolute Deviations Estimators for Censored Regression

Suppose random variables $y_1, y_2, \dots$ are generated by a regression $y_i = x_i'\theta_0 + u_i$, with θ_0 an unknown d-dimensional vector of parameters, $\{x_i\}$ a sequence of observed vectors, and $\{u_i\}$ unobserved errors. The method of least absolute deviations would estimate θ_0 by the θ that minimized the convex function

$$\sum_{i \le n} |y_i - x_i'\theta|.$$

Convexity in θ makes the asymptotic analysis not too difficult (Pollard 1990). Much more challenging is a related problem, analyzed by Powell (1984), in which the value of y_i is observed only if $y_i \ge 0$ and otherwise only the information that $y_i < 0$ is available. That is, only y_i^+ is observed. In the econometrics literature this is called a Tobit model (at least when the $\{u_i\}$ are independent normals).

Powell proposed an interesting variation on the least absolute deviations estimation; he studied the $\widehat{\theta}_n$ that minimizes

$$\sum_{i \le n} |y_i^+ - (x_i'\theta)^+|$$

over a subset Θ of $\mathbb{R}^d$. This function is not convex in θ; analysis of $\widehat{\theta}_n$ is quite difficult. However, by extending a technique due to Huber (1967), Powell was able to give conditions under which $\sqrt{n}(\widehat{\theta}_n - \theta_0)$ has an asymptotic normal distribution.

With the help of the maximal inequalities developed in these notes, we can relax Powell's assumptions and simplify the analysis a little. The strategy will be to develop a uniformly good quadratic approximation to the criterion function, then show that $\widehat{\theta}_n$ comes close to maximizing the approximation. Powell's consistency argument was based on the same idea, but for asymptotic normality he sought