## SECTION 5

# Stability

Oftentimes an interesting process can be put together from simpler processes, to which the combinatorial methods of Section 4 apply directly. The question then becomes one of stability: Does the process inherit the nice properties from its component pieces? This section provides some answers for the case of processes $\boldsymbol{\sigma} \cdot \mathbf{f}$ indexed by subsets of Euclidean space.

Throughout the section $\mathcal{F}$ and $\mathcal{G}$ will be fixed subsets of $\mathbb{R}^n$, with envelopes $\mathbf{F}$ and $\mathbf{G}$ and $\boldsymbol{\sigma} = (\sigma_1, \ldots, \sigma_n)$ will be a vector of independent random variables, each taking the values $\pm 1$ with probability $1/2$. In particular, $\boldsymbol{\sigma}$ will be regarded as the generic point in the set $\mathcal{S}$ of all $n$-tuples of $\pm 1$'s, under its uniform distribution $\mathbb{P}_\sigma$. The problem is to determine which properties of $\mathcal{F}$ and $\mathcal{G}$ are inherited by classes such as

$$\begin{aligned}
\mathcal{F} \oplus \mathcal{G} &= \{\mathbf{f} + \mathbf{g} : \mathbf{f} \in \mathcal{F}, \mathbf{g} \in \mathcal{G}\}, \\
\mathcal{F} \vee \mathcal{G} &= \{\mathbf{f} \vee \mathbf{g} : \mathbf{f} \in \mathcal{F}, \mathbf{g} \in \mathcal{G}\}, \\
\mathcal{F} \wedge \mathcal{G} &= \{\mathbf{f} \wedge \mathbf{g} : \mathbf{f} \in \mathcal{F}, \mathbf{g} \in \mathcal{G}\}, \\
\mathcal{F} \odot \mathcal{G} &= \{\mathbf{f} \odot \mathbf{g} : \mathbf{f} \in \mathcal{F}, \mathbf{g} \in \mathcal{G}\}.
\end{aligned}$$

The reader might want to skip the material in the subsection headed "General Maximal Inequalities". It is included in this section merely to illustrate one of the more recent developments in the subject; it is based on the paper by Ledoux and Talagrand (1989). For most applications to asymptotic problems, the simpler results contained in the first two subsections seem to suffice.

**Pseudodimension.** This property is stable only for the formation of unions, pointwise maxima, and pointwise minima.

Suppose that both $\mathcal{F}$ and $\mathcal{G}$ have pseudodimension at most $V$. Then, for every $\mathbf{t}$ in $\mathbb{R}^k$ and every $k$ less than $n$, Lemma 4.6 asserts that the projection of $\mathcal{F}$ can occupy at most

$$m = \binom{k}{0} + \cdots + \binom{k}{V}$$