SECTION 1

Introduction

As it has developed over the last decade, abstract empirical process theory has largely been concerned with uniform analogues of the classical limit theorems for sums of independent random variables, such as the law of large numbers, the central limit theorem, and the law of the iterated logarithm. In particular, the Glivenko-Cantelli Theorem and Donsker's Theorem, for empirical distribution functions on the real line, have been generalized and extended in several directions. Progress has depended upon the development of new techniques for establishing maximal inequalities for sums of independent stochastic processes. These inequalities can also be put to other uses in the asymptotic theory of mathematical statistics and econometrics. With these lecture notes I hope to explain some of the theoretical developments and illustrate their application by means of four nontrivial and challenging examples.

The notes will emphasize a single method that has evolved from the concept of a Vapnik-Červonenkis class of sets. The results attained will not be the best possible of their kind. Instead I have chosen to strive for just enough generality to handle the illustrative examples without having to impose unnatural extra conditions needed to squeeze them into the framework of existing theory.

Usually the theory in the literature has concerned independent (often, also identically distributed) random elements $\xi_1, \xi_2, \ldots$ of an abstract set Ξ. That is, for some σ-field on Ξ, each ξ_i is a measurable map from a probability space $(\Omega, \mathcal{A}, \mathbb{P})$ into Ξ. For each n, the $\{\xi_i\}$ define a random probability measure on the set Ξ: the *empirical measure* P_n puts mass $1/n$ at each of the points $\xi_1(\omega), \ldots, \xi_n(\omega)$. Each real-valued function f on Ξ determines a random variable,

$$P_n f = \frac{1}{n} \sum_{i \leq n} f(\xi_i(\omega)).$$

For fixed f, this is an average of independent random variables, which, under appropriate regularity conditions and with the proper standardizations, will satisfy a law of large numbers or a central limit theorem. The theory seeks to generalize these classical results so that they hold uniformly (in some sense) for f ranging