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An Optimality Property of Bayes' Test Statistics

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Dedicated to Erich Lehmann on his 90th Birthday

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Preface

This paper dates back to the late 60's when I collaborated with Raj Bahadur, who is unfortunately no longer with us. The reason it has not appeared until now is that he felt it had to be accompanied by a number of multivariate examples. We both went on to other things; the examples were not worked out although we both knew of the existence of some of them. So why is this paper appearing here (with the approval of Steve Stigler, an executor of the Bahadur estate)? First, in addition to attesting to Erich's continued vital presence, it gives me the opportunity of paying a tribute to Bahadur, who was a friend of both of ours. Second, it is an interesting reminder of how writing styles have changed on the whole I think for the better – from rigorous abstract formulation and mathematically rigorous presentation to more motivation and a lot of hand waving. Third, and most importantly, the result is an example of what I think both Erich and I consider an important endeavor, the reconciliation of the Bayesian and frequentist points of view (in context of now rather unfamiliar asymptotics). In an important paper in the 5^{th} Berkeley Symposium [4], Bahadur showed that the maximum likelihood ratio statistic possessed an optimality property from the view of a large deviation based frequentist comparison of tests he introduced in 1960 [1]. Our paper shows that this property is shared by Bayes test statistics for reasonable priors and conjectures that a corresponding Bayesian optimality property holds for the maximum likelihood ratio statistic. If true this can be viewed as the large deviation analogue of the well-known Bernstein von Mises' theorem – see Lehmann and Casella [10] p. 489, which establishes the equivalence at the $n^{-\frac{1}{2}}$ scale of Bayesian and maximum likelihood estimates. Establishing this conjecture is left as a challenge to the reader.

[1]On leave from University of California, Berkeley (1965-66).
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