Chapter 18

**GEOMETRIC 2-MANIFOLDS**

The concept “two-dimensional manifold” or “surface” will not be associated with points in three-dimensional space; rather it will be a much more general abstract idea. — Hermann Weyl (1913)

There is clearly a large variety of different surfaces around in our experiential world. The study of the geometry of general surfaces is the subject of differential geometry. In this chapter we will study geometric 2-manifolds, that is, a connected space that locally is isometric to either the (Euclidean) plane, a sphere, or a hyperbolic plane. The surface of a cylinder (no top or bottom and indefinitely long) and a cone (with the cone point removed) are examples of geometric 2-manifolds. We study these because their geometry is simpler and closely related to the geometry we have been studying of the plane, spheres, and hyperbolic planes.

There are no prerequisites for this chapter from after Chapter 7, but some of the ideas may be difficult the first time around. Problems **18.1, 18.3, and 18.6** are the minimum that is needed from this chapter before you study Chapter 24 (3-Manifolds — Shape of Space); the other problems can be skipped.

We use the term “manifold” here instead of “surface” because we usually think of surfaces as sitting extrinsically in 3-space. Here we want to study only the intrinsic geometry; and thus, any particular extrinsic embedding does not matter. Moreover, we will study some geometric 2-manifolds (for example, the flat torus) that cannot be (isometrically) embedded in 3-space. We ask: what is a two-dimensional bug’s intrinsic geometric experience on geometric 2-manifolds? How will the bug view geodesics (intrinsically straight lines) and triangles? How can a bug on a geometric 2-manifold...