CONVEX-ORDERING AMONG FUNCTIONS, WITH
APPLICATIONS TO RELIABILITY AND MATHEMATICAL
STATISTICS

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Hardy, Littlewood, and Pólya (1952) introduced the notion of one function being convex with respect to a second function and developed some inequalities concerning the means of the functions. We use this notion to establish a partial order called convex-ordering among functions. In particular, the distribution functions encountered in many parametric families in reliability theory are convex-ordered. We have formulated some inequalities which can be used for testing whether a sample comes from $F$ or $G$, when $F$ and $G$ are within the same convex-ordered family. Performance characteristics of different coherent structures can also be compared with respect to this partial ordering. For example, we will show that the reliability of a $k+1$-out-of-$n$ system is convex with respect to the reliability of a $k$-out-of-$n$ system.

When $F$ is convex with respect to $G$, the tail of the distribution $F$ is heavier than that of $G$; therefore, our convex-ordering implies stochastic ordering. Convex-ordering is also related to total positivity and monotone likelihood ratio families. This provides us a tool to obtain some useful results in reliability and mathematical statistics.

1. Introduction. Notions of partial ordering among survival distributions have played a useful role in providing numerous inequalities in reliability. The notion of a random variable $X$ with distribution $F$ being stochastically larger than another random variable $Y$ with distribution $G$ is well known in the literature. Van

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