

Contents

Preface	vii
Acknowledgments	ix
Chapter 1. Surfaces	1
1.1. Triangulating surfaces	1
1.2. Hyperbolic surfaces	6
Chapter 2. Stable commutator length	13
2.1. Commutator length and stable commutator length	13
2.2. Quasimorphisms	17
2.3. Examples	20
2.4. Bounded cohomology	26
2.5. Bavard's Duality Theorem	34
2.6. Stable commutator length as a norm	36
2.7. Further properties	41
Chapter 3. Hyperbolicity and spectral gaps	51
3.1. Hyperbolic manifolds	51
3.2. Spectral Gap Theorem	56
3.3. Examples	60
3.4. Hyperbolic groups	64
3.5. Counting quasimorphisms	71
3.6. Mapping class groups	77
3.7. $\text{Out}(F_n)$	83
Chapter 4. Free and surface groups	87
4.1. The Rationality Theorem	87
4.2. Geodesics on surfaces	110
4.3. Diagrams and small cancellation theory	127
Chapter 5. Irrationality and dynamics	137
5.1. Stein–Thompson groups	137
5.2. Groups with few quasimorphisms	143
5.3. Braid groups and transformation groups	154
Chapter 6. Combable functions and ergodic theory	163
6.1. An example	163
6.2. Groups and automata	172
6.3. Combable functions	174
6.4. Counting quasimorphisms	181

6.5. Patterson-Sullivan measures	184
Bibliography	197
Index	205