

## Preface

This anthology of six papers is a fruitful outcome of the workshop “Combinatorial Aspect of Integrable Systems”, which was held at RIMS (Research Institute for Mathematical Sciences), Kyoto University during July 26–30, 2004, as a part of the Project Research 2004 “Method of Algebraic Analysis in Integrable Systems”.

The authors were participants of the workshop and played active roles as lecturers on the subjects discussed in their papers. The following is a brief summary of them.

Prof. Berenstein with Prof. Kazhdan gives a brief survey on geometric and unipotent crystals. They are the notions introduced by the authors as algebro-geometric analogues of crystals and crystal bases. They indeed construct a functor from the category of positive geometric crystals to that of crystals.

Prof. Lecouvey’s paper is an exposition of the combinatorics of crystal graphs. Beginning with a concise review of semi-standard tableaux, bumping procedures, plactic relations and Robinson-Schensted type correspondence, it explains how these standard apparatus for type  $A$  can naturally be extended to the root systems of types  $B_n, C_n, D_n$  and  $G_2$  along a unified perspective of the crystal basis theory.

The third paper, by one of the editors, is an exposition of the so-called  $X = M$  conjecture, which has been one of the central theme in the combinatorial aspect of integrable systems in recent years. It starts with the definition of crystals basis, the combinatorial  $R$  and the energy function. The basic data on Kang-Kashiwara-Misra crystals for non-exceptional types is included. Then the notion of path is introduced, which leads to the definition of the one dimensional sum  $X$ . It is a crystal theoretical reformulation of the one appearing in Baxter’s corner transfer matrix method. Finally the fermionic formula  $M$  is introduced and the  $X = M$  conjecture is stated.

Prof. Schilling’s paper is a survey of the proof of the  $X = M$  conjecture for type  $A$ . It also contains an exposition of how the fermionic formula  $M$  arises from the Bethe ansatz on a simplest example. The proof of the  $X = M$  conjecture is done by introducing the combinatorial object called rigged configuration and establishing the bijection between them and the paths. The strategy, originally going back to Kerov-Kirillov-Reshetikhin, has been significantly refined and developed recently combined with the crystal basis theory. The paper also covers several topics from these latest developments.

Prof. Takagi’s paper is a survey of the soliton cellular automata associated with crystal bases. They are solvable lattice models at  $q = 0$ , which may be viewed as integrable systems in combinatorics. This is a place where the crystal basis theory has found another very efficient application recently. A few basic examples of such automata had been invented by intuition and known as the box-ball system, before the connection with the crystal basis theory was recognized. The paper explains how the box-ball systems can be reconstructed and systematically generalized by the crystal basis theory. It also illustrates the basic properties of the automata such as scattering rules and color separation along type  $A$  and  $D$  cases.

Prof. Veselov’s paper is a review on recent developments in the theory of the Yang-Baxter maps from the viewpoint of discrete dynamics. It covers the historical background and fundamental concepts in the theory of Yang-Baxter maps in comparison with quantum Yang-Baxter equation. Also, the question of integrability in discrete dynamics of Yang-Baxter maps is discussed in connection with various topics, such as matrix factorization, Poisson Lie groups, and geometric crystals, in order to clarify the internal relationship among apparently different subjects.

All the papers are friendly written and readable independently, and yet they are closely related to each other. We hope the readers enjoy the symphony of six movements on combinatorial aspect of integrable systems.

Atsuo Kuniba  
Masato Okado