Lecture Notes in Logic 4

Arnold W. Miller

Descriptive Set Theory and Forcing

Springer

Editorial Policy

§1. Submissions are invited in the following categories:

i) Research monographs
ii) Lecture and seminar notes
iii) Reports of meetings
iv) Texts which are out of print.

Those considering a project which might be suitable for the series are strongly advised to contact the publisher or the series editors at an early stage.

§2. Categories i) and ii). These categories will be emphasized by Lecture Notes in Logic and are normally reserved for works written by one or two authors. The goal is to report new developments – quickly, informally, and in a way that will make them accessible to non-specialists. In the evaluation of submissions timeliness of the work is an important criterion. Texts should be well-rounded and reasonably self-contained. In most cases the work will contain results of others as well as those of the authors. In each case the author(s) should provide sufficient motivation, examples, and applications. In this respect, articles intended for a journal and Ph.D. theses will usually be deemed unsuitable for the Lecture Notes series. Proposals for volumes in this category should be submitted (preferably in duplicate) either to one of the series editors or to Springer-Verlag, Heidelberg, and will be refereed. A provisional judgment on the acceptability of a project can be based on partial information about the work: a detailed outline describing the contents of each chapter, the estimated length, a bibliography, and one or two sample chapters – or a first draft. A final decision whether to accept will rest on an evaluation of the completed work which should include

- at least 100 pages of text;
- a table of contents;
- an informative introduction perhaps with some historical remarks which should be accessible to readers unfamiliar with the topic treated;
- a subject index.

§3. Category iii). Reports of meetings will be considered for publication provided that they are both of exceptional interest and devoted to a single topic. In exceptional cases some other multi-authored volumes may be considered in this category. One (or more) expert participants will act as the scientific editor(s) of the volume. They select the papers which are suitable for inclusion and have them individually refereed as for a journal. Papers not closely related to the central topic are to be excluded. Organizers should contact Lecture Notes in Logic at the planning stage.

§4. Category iv). This category provides an avenue whereby books which have gone out of print but which are still in demand can be made available to the new generations of logicians.

§5. Format. Works in English are preferred. They should be submitted in camera-ready form according to Springer-Verlag's specifications. Technical instructions and/or TeX macros will be sent on request.

Lecture Notes in Logic 4

Springer
Berlin
Heidelberg
New York
Barcelona
Budapest
Hong Kong
London
Milan
Paris
Tokyo

Arnold W. Miller

Descriptive Set Theory and Forcing

How to prove theorems about Borel sets the hard way

Springer

Author

Arnold W. Miller
Department of Mathematics
480 Lincoln Drive
Van Vleck Hall
University of Wisconsin
Madison, WI 53706, USA
E-mail: miller@math.wisc.edu

Cataloging-in-Publication Data applied for

Die Deutsche Bibliothek - CIP-Einheitsaufnahme

Miller, Arnold W.:
Descriptive set theory and forcing : how to prove theorems about Borel sets the hard way / Arnold W. Miller. - Berlin ; Heidelberg ; New York ; Barcelona ; Budapest ; Hong Kong ; London ; Milan ; Paris ; Tokyo : Springer, 1995
(Lecture notes in logic ; 4)
ISBN 3-540-60059-0 (Berlin ...)
ISBN 0-387-60059-0 (New York ...)
NE: GT

Mathematics Subject Classification (1991): 03E15, 03E35

ISBN 3-540-60059-0 Springer-Verlag Berlin Heidelberg New York

Printed in Germany

Typesetting: Camera-ready by the authors
SPIN: 10126442 2146/3140-543210 - Printed on acid-free paper

Note to the readers

Departing from the usual author's statement-I would like to say that I am not responsible for any of the mistakes in this document. Any mistakes here are the responsibility of the reader. If anybody wants to point out a mistake to me, I promise to respond by saying "but you know what I meant to say, don't you?"

These are lecture notes from a course I gave at the University of Wisconsin during the Spring semester of 1993. Some knowledge of forcing is assumed as well as a modicum of elementary Mathematical Logic, for example, the Lowenheim-Skolem Theorem. The students in my class had a one semester course, introduction to mathematical logic covering the completeness theorem and incompleteness theorem, a set theory course using Kunen [54], and a model theory course using Chang and Keisler [17]. Another good reference for set theory is Jech [43]. Oxtoby [88] is a good reference for the basic material concerning measure and category on the real line. Kuratowski [57] and Kuratowski and Mostowski [58] are excellent references for classical descriptive set theory. Moschovakis [87] and Kechris [52] are more modern treatments of descriptive set theory.

The first part is devoted to the general area of Borel hierarchies, a subject which has always interested me. The results in section 14 and 15 are new and answer questions from my thesis. I have also included (without permission) an unpublished result of Fremlin (Theorem 13.4).

Part II is devoted to results concerning the low projective hierarchy. It ends with a theorem of Harrington from his thesis that is consistent to have $\undertilde{\Pi}^1_2$ sets of arbitrary size.

The general aim of part III and IV is to get to Louveau's theorem. Along the way many of the classical theorems of descriptive set theory are presented "just-in-time" for when they are needed. This technology allows the reader to keep from overfilling his or her memory storage device. I think the proof given of Louveau's Theorem 33.1 is also a little different. [1]

Questions like "Who proved what?" always interest me, so I have included my best guess here. Hopefully, I have managed to offend a large number of mathematicians.

[1] In a randomly infinite Universe, any event occurring here and now with finite probability must be occurring simultaneously at an infinite number of other sites in the Universe. It is hard to evaluate this idea any further, but one thing is certain: if it is true then it is certainly not original!– The Anthropic Cosmological Principle, by John Barrow and Frank Tipler.

Contents