Part II

Contributed Papers

Gödel's Ontological Proof Revisited *

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Gödel's version of the modal ontological argument for the existence of God has been criticized by J. Howard Sobel [5] and modified by C. Anthony Anderson [1]. In the present paper we consider the extent to which Anderson's emendation is defeated by the type of objection first offered by the Monk Gaunilo to St. Anselm's original Ontological Argument. And we try to push the analysis of this Gödelian argument a bit further to bring it into closer agreement with the details of Gödel's own formulation. Finally, we indicate what seems to be the main weakness of this emendation of Gödel's attempted proof.

1.

Gaunilo observed against St. Anselm that his form of argument, if cogent, could be used to "prove" all sorts of unwelcome conclusions — for example, that there is somewhere a perfect island. It would seem to even follow that there are near-perfect, but defective, demi-gods and all matter of other theologically repugnant entities. Gaunilo concluded, reasonably enough, that something must be wrong with the argument.

Kurt Gödel's modern version of the Ontological Argument [12] involves an attempt to complete the details of Leibniz's proof that it is possible that there is a perfect being or a being with all and only "positive" attributes. Given this conclusion, other assumptions about positive properties, and, well, a second-order extension of the modal logic S5, Gödel successfully deduced the actual existence, indeed the necessary existence, of the being having all and only positive attributes. Alas, or "Oh, joy!", depending on ones' theological prejudices, J. Howard Sobel showed that Gödel's assumptions lead also to the conclusion that whatever is true is necessarily true. Followers of Spinoza aside, this casts quite considerable doubt on the premisses of the argument. We shall consider here Anderson's emendation which does not suffer from the mentioned defect and which is still recognizably closely related to Gödel's argument. [1]

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* This paper is in its final form and no similar paper has been or is being submitted elsewhere.

[1] Petr Hájek [13] has argued that Anderson's version of the argument has superfluous premisses, but the truth of this claim depends on the details of the underlying second-order modal logic adopted. In the context of the (quite reasonable) version of that logic formulated by Nino Cocchiarella [2] and cited by

Here are the assumptions and definitions — the notion of a positive attribute is taken as a primitive by Gödel and in the present version. We hasten to add that the idea is not crystal clear; Gödel's own explanations are extremely terse and somewhat cryptic. A property's being positive is supposed to be a good thing, such properties being characteristic of a completely and necessarily non-defective being.

(A1*) If Φ is positive, then its complement non-Φ is not positive.

(A2*) If Φ is positive and necessarily all Φ's are Ψ's, then Ψ is positive — that is, properties entailed by positive properties are themselves positive.

Definition: "x is Godlike" means by definition that x has a property Φ necessarily, if and only if the property Φ is positive.

(A3*) Godlikeness is a positive property.

(A4*) If a property Φ is positive, then it is necessarily positive.

Definition: "Property Φ is an essence of an entity x" means that Φ entails all and only those properties which x has necessarily.

Definition: aa$x$ is necessarily existent" means that every essence of x is necessarily instantiated.

(A5*) Necessary existence is a positive property.

From these it follows (in the mentioned logic) that it is necessary that there exists a Godlike being — indeed that such a being is unique. (Details of the argument may be found in Anderson [1].)

What would Gaunilo say? Following a suggestion of Patrick Grim's, he might argue thus: Let's say that a property is "restricted-positive" if it is positive, but does *not* entail some particular positive property — say moral goodness. Now define x to be "nearly-Godlike" if it has a property necessarily, if and only if that property is *restricted*-positive. Can we not argue, heretically, that there necessarily exists a nearly Godlike being — a being otherwise perfect, but lacking moral perfection?

Curiously, the objection fails. We find this a bit surprising since Gaunilo-type objections seem to apply powerfully and persuasively to virtually every other version of the Ontological Argument with which we are acquainted.

To produce a persuasive reductio, the Gaunilist parallel argument must make the corresponding assumptions in his Near-Ontological Argument. In the present case, he must assume, in addition to the analogues of the other axioms, the analogue of axiom (A3*), namely:

Anderson as the underlying logic, Hájek's point does *not* hold. It does indeed follow from the diminished set of premisses that: if THERE IS a Godlike being, then necessarily THERE IS such a being. From this one can easily deduce that THERE IS a Godlike being, as Hajek observes. But the sense of the quantifier indicated by the capitalized phrase here (i.e., as formalized in Cocchiarella's logic) is that of "possible existence" or "subsistence". To express actual existence requires a separate quantifier. So without using the other premisses, nothing yet follows about the actual existence of a Godlike being.

(N3*) Near-godlikeness is a restricted-positive property.

But this won't do. Use "G'" for the property of near-Godlikeness and attend to the definition of this property and of restricted-positivity. Since moral goodness, call it M, is positive and entails the disjunctive property of being either non-G'-or-M, this latter property must be positive. [2] Now this disjunctive property does not itself entail M (assuming, what is evident, that the property of being non-G' is possibly exemplified). Hence the disjunctive property non-G'-or-M is restricted-positive. Given the definition of near-Godlikeness (viz., having all and only restricted-positive properties necessarily), it follows that near-Godlikeness, G', entails the disjunctive property non-G'-or-M. Hence G' entails M — and so is *not* itself restricted-positive, contrary to (N3*).

Of course the Gaunilist might find some defect in the axiom we have used against him — that properties entailed by a positive property are themselves positive. But this is very plausibly construed as a crucial feature of the very idea of positiveness to which, obscure as it is, the Ontological Arguer is entitled. There is also the further possibility that some other Gaunilo-type argument will succeed where this one has failed, but our best efforts to produce such have so far been fruitless. We conclude, tentatively, that this emendation of Gödel's Ontological Proof may well be immune to the Gaunilo-type of objection.

2.

We suggest further revisions of Anderson's modifications of Gödel's Ontological Argument which bring the assumptions and reasoning closer to Gödel's original intent. In the place of the Axiom A1 (and Anderson's modified A1*), we propose:

(A1') $\neg[Pos(\Box F) \equiv Pos(\neg \Box F)]$,

that is, exactly one of the two properties *being necessarily F* and *not being necessarily F* is positive. (We use obvious abbreviations here. For example, '$\Box F$' abbreviates '$\lambda x \Box F(x)$' — the property which anything x has when it is necessarily F.) Gödel's corresponding axiom asserts that of each property and its negation, or complement, exactly one is positive. This can be seen as in some measure responsible for the "modal collapse" noted by Sobel — that is, from Gödel's assumptions and using a natural second-order modal logic, it follows that every proposition which is true is necessary. Anderson's emendation replaced Gödel's premiss by its weaker consequence: if a property is positive, then its negation is not. The present axiom seems closer in spirit to

[2] The Gaunilist has offered no objection to axiom (A2*) and indeed may use it to prove his analogue thereof.

the original, since it just modalizes the property before asserting the exclusive disjunction.

A second alteration we propose is to eliminate what is apparently Dana Scott's gloss on one of Gödel's premisses in favor of that exact assumption (See Sobel [5]). Scott simplified Gödel's argument by assuming that *being Godlike* is positive and Anderson paralleled this with the premiss that *being Godlike** is positive. But in the original note, Gödel had instead that the conjunction of any number of positive properties is positive, including the "infinite conjunction" formed by universal quantification. Since to be Godlike is to have all and only positive properties, Scott's premiss follows easily from Gödel's definition and the Conjunction Assumption (as we shall call it). So we adopt:

(A3') If F is any set of positive properties, then the property obtained by taking the conjunction of the properties in F is positive. (One needs third-order logic to state this formally. In that context, the assumption is:

(A3') $\forall F[\Phi(F) \supset Pos(F)] \supset \forall G \, \Box \forall x\{G(x) \equiv \forall F[\Phi(F) \supset F(x)]\} \supset Pos(G)$.)

Finally, we adopt an assumption which Gödel explicitly endorses elsewhere in his notes on the Ontological Argument (Cf. [12]): that the necessitation of a positive property is positive:

(A6') $Pos(F) \supset Pos(\Box F)$.

Gödel observes that with this addition the assumption (A5) that Necessary Existence is a positive property can be replaced by the weaker premiss that Existence itself is a positive property. (Here, to avoid needless controversy, we can take Existence to be defined as follows: x exists if and only if some essence F of x is exemplified, i.e. there is a y such that $F(y)$).

It is interesting to observe that with these assumptions and utilizing Anderson's emended definitions of "essence" and "Godlikeness", we can deduce the conclusion that necessarily there exists a Godlike being. As presently advised, then, we propose as a rational reconstruction of Gödel's reasoning, the assumptions stated herein, together with A2, and A4. In addition we still maintain that the emended definitions of essence and Godlikeness are preferable for the present purposes. In an appendix, and on the handout, we sketch proofs of the technical claims we have made.

3.

Does Gödel's Ontological Argument then rest secure? We think not. Anderson and, especially, Robert Adams have emphasized that taking the conjunction of two or more positive properties to be positive or else just assuming that

Godlikeness is positive, together with the assumption that positive properties entail only positive properties comes quite close, epistemically, to just assuming that the existence of God (defined in Gödel's or in Anderson's way) is possible. We can of course demand that a positive property be *purely* positive — it must entail no negation, and in particular, no contradiction. But then how can we be sure that taking conjunctions of such will not yield an impossibility (given that the incompatibility involved need not be purely "formal").

Leibniz thought that the premiss that God's existence is possible needs to be proved and he attempted such a proof using the idea that a perfection (which corresponds roughly to Gödel's notion of a positive property) must be "simple". And he thought that it is then reasonable to conclude that the conjunction of two or more perfections cannot be impossible. But it just isn't clear that this is so and if we make assumptions that guarantee that it is so, along the lines of Gödel's axioms, then it is difficult to see that any epistemic progress has been made. We suggest that the Gödelian Ontological Arguer should simply admit that neither the possibility of God nor the truth of the axioms used to "prove" that possibility are self-evident. And he might just maintain that the less evident axioms, for example that a conjunction of positive properties is positive, is an assumption which he adopts on grounds of mere plausibility and is entitled to accept until some incompatibility between clearly positive properties is discovered.

Appendix

Assuming A1', we show that
(1) $\neg Pos(F) \supset Pos(\neg \Box F)$.

Proof. Assume that F is not positive. Then necessarily F cannot be positive, for necessarily F entails F — and the properties entailed by a positive property are positive (A2). Hence the second alternative of A1' must hold: $Pos(\neg \Box F)$.

Now contemplate the definition of Godlikeness* — something is Godlike* if and only if it has all and only positive properties essentially. We show, using Gödel's general "conjunction axiom" (A3'), that Godlikeness* is positive. To do so, it is evidently sufficient to show that any property of the form $\lambda x\{\Box H(x) \equiv Pos(H)\}$ is positive — since Godlikeness* is the "infinite conjunction" of them. In turn, to show this, it suffices to show that each property of the form: $\lambda x\{\Box H(x) \supset Pos(H)\}$ and each property of the form $\lambda x\{Pos(H) \supset \Box H(x)\}$ is positive.
(2) $\lambda x\{\Box H(x) \supset Pos(H)\}$ is always a positive property.

Proof. Either H is positive or it is not. If so, then the indicated property is positive, being entailed by the (vacuous) and necessary property $\lambda x Pos(H)$

((A4), (A2)). If it is not, then by (1), $Pos(\neg \Box H)$ — and the indicated property is entailed by $\neg \Box H$ (Again use A2).

(3) $\lambda x\{Pos(H) \supset \Box H(x)\}$ is always a positive property.

Proof. If $Pos(H)$, then $Pos(\Box H)$ by A6 — and the indicated property is entailed by $\Box H$ (A2).

We may conclude from (2), (3) and the conjunction axiom (A3') that Godlikeness* is positive — Anderson's original Axiom A3*.

We note further that Anderson's original modification of Gödel's first axiom is a consequence of the present set of assumptions:
(4) $Pos(F) \supset \neg Pos(\neg F)$.

Proof. Assume that F is positive and, for a reductio, that $\neg F$ is positive as well. Then by (A6), $Pos(\Box F)$. Furthermore, if $\neg F$ is positive, then so is $\Diamond \neg F$, being entailed by it. But this latter is just the property $\neg \Box F$, so that we have both $Pos(\Box F)$ and $Pos(\neg \Box F)$ — contradicting A1'.

The line of reasoning Gödel suggests for proving that Necessary Existence is positive, from the premiss that Existence is positive may be reproduced here if we take as our definition of Existence: $E(x) =_d f\forall F[FEssx \supset \exists x F(x)]$. [If this reasoning is formalized using Cocchiarella's system, the existential quantifier here is actualist.]

Thus the reasoning of Anderson's version of the Gödel's Ontological Argument goes through, given the present premisses.

References

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