

WILLIAM FELLER, *IN MEMORIAM*

William Feller, one of the most original, accomplished, and colorful mathematicians of our times, died after a long illness on 14 January, 1970. The entire mathematical community mourns his death but at this symposium his loss will be felt more deeply, for all of us here have been influenced both by his work and by his person.

Feller's contributions to probability theory are so well known to, and appreciated by, everyone in this audience that reviewing them is hardly necessary. If I, nevertheless, engage in a brief recital of his accomplishments it is mainly to remind us of how central his role was and how much our subject owes to him.

Except for a few papers on foundations (two written jointly with Tornier and containing first hints of applications of probabilistic methods to number theory) Feller's earlier work is devoted to the most classical problems of probability theory.

By showing that Lindeberg's sufficient conditions are also necessary for the central limit theorem to hold and by providing necessary and sufficient conditions for the weak law of large numbers, Feller completes that chapter of our subject which was begun by Bernoulli, De Moivre and Laplace.

A little later he immediately recognizes the importance of Kolmogorov's 1931 paper "Über die analytischen Methoden in der Wahrscheinlichkeitsrechnung" and provides a most significant continuation in "Zur Theorie der stochastischen Prozesse" (1936), in which his mastery of differential equations acquired and nurtured in Göttingen is brilliantly displayed.

In 1939 Feller comes to the United States and, as if to celebrate this event, he publishes a few years later his famous memoir on the law of the iterated logarithm—a veritable *tour de force* and a theme with an infinitude of variations, some of which he invents and plays even during the last days of his life.

In 1950 at the age of forty-four, when according to legend most mathematicians are supposed to be long past the period of real creativity, Feller embarks on an entirely new line of inquiry.

Going back to his earlier work on diffusion theory, he now brings to bear on this classical subject and its extensions the power of modern functional analysis with striking results. In his hands semigroup theory reaches new depths and at the same time a subject as old and as seemingly exhausted as the Sturm-Liouville theory acquires a glitter of new life. In a sense it is Feller's most original work and it is soon continued and extended all over the world.

The five year period from 1945 to 1950, which just preceded the beginning of Feller's work in semigroups and Markov processes, is largely devoted to writing Volume I, although it is during this period that he also publishes, among others, his elegant and influential memoir on "Fluctuation Theory of Recurrent Events."

Volume I, or to be precise, *An Introduction to Probability Theory and its Applications*, Volume I, is a book with few peers in scientific literature. It is a treatise and a textbook, a masterpiece of exposition and a credo of methodology, a sweeping panorama of a subject and a collection of exemplary jewels. No wonder it has appealed to an audience so wide as to border on the incredible, no wonder that no other book on the subject, not even Volume II, which appeared in 1966, can match its luster.

But all Feller's achievements do not end here. In the true Göttingen tradition he keeps up a lively interest in applications, especially to population genetics and the theory of evolution. His semi-expository paper "Diffusion processes in genetics" in the *Proceedings of the Second Berkeley Symposium* brings to the attention of mathematicians and statisticians the beautiful ideas of Sewall Wright and R. A. Fisher, and marks the beginning of much subsequent work on the interplay of diffusion and branching processes.

Feller continues to be deeply involved with problems of genetics to the end of his life, and as Permanent Visiting Professor at The Rockefeller University he develops (and greatly enjoys!) a close collaboration with Professor Dobzhansky and his group.

Feller's work was widely recognized and appreciated. He was elected a member of the National Academy of Sciences, the American Academy of Arts and Sciences, and of the American Philosophical Society. He was a foreign member of the Danish and Yugoslav Academies and he served as President of the Institute of Mathematical Statistics.

The highest honor came almost too late.

Just before he entered the hospital for the last time he learned that he had been awarded the Medal of Science for 1970. He did not live to attend the White House ceremony and it was his widow, Clara, who accepted the medal from the President of the United States.

Feller was a man of enormous vitality. Not even in the last stages of his illness was his zest for life visibly lessened. The intensity of his reactions was reflected in what his friends called the "Feller factor," an imprecisely defined number by which one had to scale down some of his pronouncements to get near the truth.

He held opinions with such strength that not infrequently they got in the way of facts with results which were more often amusing than painful. But he was not stubborn and, underneath the bluster, kind and generous.

Much as he loved mathematics, his view of it was anything but parochial. As a man of wide interests and culture he knew that there are many ways in which one can be clever and he considered the notion that mathematicians are better equipped to deal with problems outside of mathematics to be, to say the least, naive.

I recall a conversation in which a colleague asked, rhetorically perhaps, "What can the generals do that we mathematicians couldn't do better?" "Sleep during battle," said Feller, and that was that.

Outside mathematics and science Feller was especially interested in ancient history—and in this fascinating field his knowledge and competence bordered on the professional.

When he learned that his illness was terminal his courage and considerateness came poignantly to the fore. Having accepted the verdict himself he tried to make it easy for all of us to accept it too. He behaved so naturally and he took such interest in things around him that he made us almost forget from time to time that he was mortally ill.

Unbelievably, he could concentrate on his work and he actually wrote up a number of papers as well as prepared a new edition of Volume II.

To the end he was as I always knew him: vital, intense, full of his own brand of exaggerations and strongly held opinions, fond of men, women, and children, and a lot of fun.

M. Kac