

**PROCEEDINGS OF THE  
THIRD BERKELEY SYMPOSIUM**

**VOLUME III**

PROCEEDINGS *of the* THIRD  
BERKELEY SYMPOSIUM ON  
MATHEMATICAL STATISTICS  
AND PROBABILITY

*Held at the Statistical Laboratory  
University of California  
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VOLUME III

CONTRIBUTIONS TO ASTRONOMY AND PHYSICS

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## ACKNOWLEDGMENT

The astronomical papers printed in this volume were delivered at the two sessions on Astronomy, December, 1954, organized with the cooperation of Professors C. D. Shane and O. Struve. Most of the remaining papers were delivered at the session on Physics, also held in December, 1954, organized with the cooperation of Professor Mina Rees and Professor William A. Nierenberg acting, respectively, for Section A of the American Association for the Advancement of Science, and for the American Physical Society.

## PREFACE

THE THIRD BERKELEY SYMPOSIUM on Mathematical Statistics and Probability was held in two parts, one from December 26 to 31, 1954, emphasizing applications, and the other, in July and August, 1955, emphasizing theory. The Symposium was thus divided because, on the one hand, it was thought desirable to provide an opportunity for contacts between American and foreign scholars who could come to Berkeley in the summer, but not in the winter, and because, on the other hand, the 121st Annual Meeting of the American Association for the Advancement of Science held in Berkeley in December, 1954, provided an opportunity for joint sessions on the various fields of applications with its many member societies. With the help of Dr. Raymond L. Taylor, of the AAAS, nine cosponsored sessions of the Symposium were organized. Two of these were given to problems of astronomy and one each to biology, medicine and public health, statistical mechanics, industrial research, psychometry, philosophy of probability, and to statistics proper.

The importance of the second part of the Symposium, which emphasized theory, was increased by the decision of the Council of the Institute of Mathematical Statistics to hold its first Summer Institute in Berkeley and to hold this Institute "in conjunction with the Third Berkeley Symposium"; all members of the IMS Summer Institute were invited to participate in the Symposium and the two enterprises were conducted in parallel. In particular, the cooperation of Professor David Blackwell, Chairman of the IMS Summer Institute, made it possible to ensure that representatives of all the major centers of statistical research in this country be invited. As will be seen from the lists of contents of the *Proceedings*, the response was good, although various circumstances, including the concurrent Rio de Janeiro meeting of the International Statistical Institute, prevented some of the prospective participants from attending the Berkeley meetings.

Two months were allotted to the second part of the Symposium in order to provide an opportunity not only for formal presentation of papers, but also for informal contacts among the participants. To facilitate such personal associations, after three weeks of intensive lectures and discussions, a trip was made to the Sierra. There, animated discussions of stochastic processes and of decision functions were interspersed with expressions of delight at the beauty of Yosemite Valley, Emerald Bay, and Feather River Canyon. After this vacation there was another period of intensive lecturing.

Although much effort was expended to arrange lectures and personal contacts, the primary concern of the Statistical Laboratory and of the Department of Statistics was with the *Proceedings*. Because of the participation of the AAAS, the amount of material submitted for publication was estimated to be equivalent to thirteen hundred printed pages, roughly twice the length of the *Proceedings of the Second Berkeley Symposium*. This presented a most embarrassing problem.

That it was finally solved is largely the result of the most effective support and advice of Dr. John Curtiss, Executive Director of the American Mathematical Society. His organizational talent and friendly help are greatly appreciated. Special thanks are due Mr. August Frugé, the Manager of the Publishing Department of the University of California Press, and also his staff, who undertook the difficult and costly publication in the best spirit of cooperation with, and of service to, the scholarly community.

Since a single thirteen-hundred-page volume would have been difficult to handle and, for the majority of scholars, too expensive to buy, it was decided to issue the *Proceedings* in five relatively small volumes, each given to a specialized and, so far as possible, unified cycle of ideas. A list of contents of the other four volumes of the *Proceedings* will be found preceding this preface.

The initial steps in the organization of the Symposium were based on a grant obtained from the University of California through the good offices of Professor Clark Kerr, Chancellor of the Berkeley campus of the University of California, to whom sincere thanks are due. This grant was followed by an appropriation from the Editorial Committee of the University of California, which provided the nucleus of the fund eventually collected for the publication of the *Proceedings*. This action of the Editorial Committee is gratefully acknowledged. For further effective support of the Symposium thanks must be given the National Science Foundation, the United States Air Force Research and Development Command, the United States Army Office of Ordnance Research, and the United States Navy Office of Naval Research. It is hoped that the material in the present *Proceedings* and, particularly, the scientific developments stimulated by the Symposium, will be sufficiently important to prove that the money received from these organizations was well spent.

The success of the Symposium was, in large part, made possible by the generous and effective support of a number of scholarly societies. Sessions of the first part of the Symposium were sponsored by the American Physical Society; the American Statistical Association; the Astronomical Society of the Pacific; the Biometric Society, Western North American Region; the Ecological Society of America; the Institute of Mathematical Statistics; the Philosophy of Science Association; and the Western Psychological Society. The American Mathematical Society sponsored the second part of the Symposium, delegating for organizational work two of its most distinguished members, Professor J. L. Doob and Professor G. Pólya, whose advice and cooperation were most helpful. The 1955 Summer Institute of the Institute of Mathematical Statistics was held in conjunction with the Symposium; the IMS also held its Western Regional Meeting in Berkeley in July.

All papers published in these *Proceedings* were written at the invitation of the Statistical Laboratory, acting either on its own initiative or at the suggestion of the cooperating groups, and the Laboratory is, therefore, responsible for the selection of the authors; a responsibility that does not extend to the contents of the papers. The editorial work on the manuscripts submitted was limited to satisfying the requirements of the University of California Press regarding the external form of the material submitted, the numbering of formulas, etc., and to correcting obvious errors in transcription. Most of this was done by the staff of the Laboratory; in particular, Miss Catherine FitzGibbon, Mrs. Jeanne Lovasich, Mrs. Kathleen Wehner, and my colleague, Dr. Elizabeth L. Scott, who supervised some of the work. Occasionally, manuscripts were read by other participants in the Symposium particularly interested in them, and the authors were offered suggestions. However, in no case was there any pressure on the authors to introduce any material change into their work.

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