Abstracts of Lectures

Caucher Birkar (University of Cambridge, UK)
Minimal model program and moduli spaces

First, I will quickly recall some of the basic elements of the minimal model program. Then, I concentrate on the interactions between the minimal model program and some moduli problems. In particular, I explain the construction of moduli spaces of varieties of general type via the minimal model program.

Ching-Li Chai (University of Pennsylvania, USA)
Modular varieties and Hecke symmetry

The Hecke symmetry is a salient feature of Shimura varieties. In the more recent past these symmetries have led to the notion of *leaves*, a global structure on the reduction of modular varieties of PEL type defined by Oort, and conjecturally for all Shimura varieties with good reduction. The Hecke symmetry also led to the notion of canonical coordinates on leaves, generalizing the classical Serre–Tate coordinates. We will discuss some recent works related to the Hecke symmetry:

(1) ℓ-adic and p-adic monodromy of leaves, and irreducibility of leaves (joint work with F. Oort). One of the methods for showing that the p-adic monodromy is big is inspired by H. Hida.
(2) CM lifting modulo Hecke symmetry (joint work with B. Conrad and F. Oort).

Christopher Deninger (Münster University, Germany)
Vector bundles and p-adic representations

In this lecture we explain how to attach a p-adic representation to a vector bundle with "strongly semistable reduction of degree zero" on a p-adic curve. We also discuss higher dimensional generalizations. This is a report on joint work with Annette Werner.

Kazuhiro Fujiwara (Nagoya University, Japan)
Galois deformation and algebraic number theory

Since the proof of Iwasawa's main conjecture in the classical case (theorem of Mazur and Wiles), automorphic forms on $GL(2)$ have played an essential role to understand $GL(1)$-problems. After the breakthrough by A. Wiles (Ann. of Math. 1995), there has been a substantial progress in the class field theory for GL_2. In this talk, open problems in classical Iwasawa theory are discussed from this perspective, based on Galois deformations.

Gerard van der Geer (University of Amsterdam, Holland)
Euler chacteristics of local systems
on the moduli of curves and abelian varieties of low genus

The cohomology of local systems on the moduli of curves and of abelian varieties carries a wealth of arithmetic information; for example it yields interesting Galois representations and modular forms. Therefore, as a first step, it is useful to be able to calculate the Euler characteristic of such local systems. This is classical for genus 1, and was done for genus 2 by Getzler. In this talk it is shown how to calculate these Euler characteristics for genus 3. (Joint work with Bini and Bergstroem.)

Klaus Hulek (Hannover University, Germany)
Intersection theory of divisors on compactifications of $\mathcal{A}_g$

We study the top intersection numbers of the boundary and Hodge class divisors on toroidal compactifications of the moduli space $\mathcal{A}_g$ of principally polarized abelian varieties and compute those numbers that live away from the stratum which lies over the closure of $\mathcal{A}_{g-3}$ in the Satake compactification. Many of these numbers turn out be 0, and we discuss a conjecture which says that this is a more general phenomenon. This is joint work with C. Erdenberger and S. Grushevsky.

Tetsushi Ito (Kyoto University, Japan)
Hasse invariants and the l-adic cohomology
of unitary Shimura varieties

The classical Hasse invariant is a modular form of weight $p-1$ in characteristic p which has a simple zero at each supersingular point. It is a fundamental tool to study the geometry of modular curves. In this talk, I will discuss how to generalize the Hasse invariant to unitary Shimura varieties with signature $(1, n-1)$ using the idea of Ekedahl–Oort stratification. Then, I will discuss an application of it to the l-adic cohomology of unitary Shimura varieties with bad reduction (Iwahori level structure) using semistable models and the weight spectral sequence of Rapoport–Zink.

Eduard Looijenga (University of Utrecht, Holland)
Invariants of certain homogeneous forms
as meromorphic automorphic forms

In a few precious instances there is a period map that establishes a correspondence as suggested by the title. For instance, the invariants of cubic forms in $n = 2, 3, 4, 5$ variables can be regarded as automorphic forms on a complex ball (of dim 1,4,10) or a 20-dim symmetric domain of orthogonal type, where for $n = 3, 4$ we must allow the automorphic forms to have prescribed polar loci. (The case $n = 2$ is one of the jewels of 19th century mathematics, but the other cases are more recent, with major contributions by Allcock–Carlson–Toledo for $n = 3, 4$.) We discuss this correspondence and the underlying Baily–Borel technology.

Vikram B. Mehta (Tata Institute, India)

The Frobenius morphism and principal G-bundles on affine varieties (joint work with S. Subramanian)

We study the application of the theory of semistable vector bundles and G-bundles to local triviality of G-bundles. This has consequences for certain conjectures in this area.

Shigeru Mukai (RIMS, Kyoto University, Japan)

Enriques surfaces associated with bielliptic abelian surfaces

Let $\overline{BP}$ be the double cover of $\mathbb{P}^1 \times \mathbb{P}^1$ with branch the union of the quadrilateral $(x^2-1)(y^2-1) = 0$ and an elliptic curve C of bidegree (2,2) passing through the four vertices. Assume that the branch possesses point symmetry. Then, taking quotient of $\overline{BP}$ by the symmetry, we obtain an Enriques surface with two rational double points of type D_4. The minimal resolution, denoted by BP, has the following remarkable property ([1, (4.8)], [3, Example 2]):

Let σ_{BP} be the involution of BP induced from the covering involution of $\overline{BP}$. Then its action on $H^2(BP, \mathbb{Z})$ is *trivial.*

We investigate this 2-dimensional family of Enriques surfaces BP using the *isogenies* of K3 surfaces studied in [2]. We will explain the following in our talk:

1. Certain correspondence, similar to Shioda–Inose[5], between $\overline{BP}$ and a bielliptic abelian surface (of even type).
2. Theorem *Every cohomologically trivial automorphism of an Enriques surface is either trivial or* σ_{BP}.
3. Another proof of the following result of Sertöz[4] as application. Theorem *A singular K3 surface (in the sense of* [6] *and* [5]*) is a K3 cover of an Enriques surface if the (rank two) transcendental lattice* T *is* very even *and* disc $T \neq 4, 8, 16$.

References

[1] W. Barth and C. Peters, Automorphism of Enriques surfaces, Invent. Math., **73** (1983), 383–411.

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[3] S. Mukai and Y. Namikawa, Automorphisms of Enriques surfaces which act trivially on the cohomology groups, Invent. Math., **77** (1984), 383–397.
[4] A. S. Sertöz, Which singular K3 surfaces cover an Enriques surface, Proc. Amer. Math. Soc., **133** (2004), 43–50.
[5] T. Shioda and H. Inose, On singular K3 surfaces, In: Complex Analysis and Algebraic Geometry, Iwanami Shoten, Tokyo, 1977, pp. 119–136.
[6] T. Shioda and N. Mitani, Singular abelian surfaces and binary quadratic forms, In: Classification of Algebraic Varieties and Compact Complex Manifolds, Lect. Notes in Math., **412**, Springer-Verlag, 1974, pp. 259–287.

Conjeeveram S. Rajan (Tata Institute, India)

On the irreducibility of irreducible characters of simple Lie algebras

We establish an irreducibility property for the characters of finite dimensional, irreducible representations of simple Lie algebras (or simple algebraic groups) over the complex numbers, i.e., that the characters of irreducible representations are irreducible after dividing out by (generalized) Weyl denominator type factors. For the factors of Weyl denominator type, the factorisation is given by the classical product expansion of the denominator of the Weyl character formula. This determines completely the factorization of characters of finite dimensional irreducible representations of complex simple algebraic groups in the algebra of regular functions of the group.

For $GL(r)$ the irreducibility result is the following: let $\lambda = (a_1 \geq a_2 \geq \cdots a_{r-1} \geq 0)$ be the highest weight of an irreducible rational representation V_λ of $GL(r)$ with trivial determinant. Assume that the integers $a_1 + r - 1, a_2 + r - 2, \cdots, a_{r-1} + 1$ are relatively prime. Then the character χ_λ of V_λ is strongly irreducible in the following sense: for any natural number d, the function $\chi_\lambda(g^d), g \in SL(r, \mathbf{C})$ is irreducible in the ring of regular functions of $SL(r, \mathbf{C})$.

Michael Rapoport (Bonn University, Germany)

Some questions on $\mathcal{G}$-bundles on curves

I will report on my joint work with G. Pappas in which we generalize the theory of algebraic loop groups and their flag varieties from the case of a split group (due to Beauville, Faltings, Laszlo, and Sorger), to the twisted case. This theory should have applications to the theory of $\mathcal{G}$-bundles on curves, just as in the split case. I will discuss the questions that arise in this context and recent progress on them.

Shigeharu Takayama (University of Tokyo, Japan)

Boundedness of pluricanonical systems
on algebraic varieties of general type

For a smooth projective variety X, the m-genus $P_m(X)$ of X for a positive integer m is defined by $P_m(X) = \dim H^0(X, \mathcal{O}_X(mK_X))$, where K_X is the canonical divisor of X. The numbers $P_m(X)$ are fundamental birational discrete invariants. The growth order of $P_m(X)$ is called the Kodaira dimension $\kappa(X)$ of X, i.e., $P_m(X) \sim m^{\kappa(X)}$ for every large and enough divisible m. It is known that we have only the following possibilities: $\kappa(X) = -\infty, 0, 1, \ldots, \dim X$. A smooth projective variety X is said to be of general type, if $\kappa(X) = \dim X$. This is equivalent to saying that there exists a positive integer m such that the pluricanonical system $|mK_X|$ gives a birational map. For X of general type, we define the volume of X by

$$\mathrm{vol}_X(K_X) := \limsup_{m \to \infty} \frac{P_m(X)}{m^n/n!},$$

where $n = \dim X$. The following theorems, due to Hacon and M$^{\mathrm{c}}$Kernan, Takayama, Tsuji, guarantee that there exist lower bounds, depending only on the dimension, for these asymptotic quantities.

Theorem 1.1. *For every positive integer n, there exists an integer m_n depending only on n such that, for every n-dimensional smooth projective variety X of general type, and for every integer $m > m_n$, the pluricanonical system $|mK_X|$ gives a birational map.*

Theorem 1.2. *For every positive integer n, there exists a positive constant v_n depending only on n such that* $\mathrm{vol}_X(K_X) \geq v_n$ *holds for every n-dimensional smooth projective variety X of general type.*

References

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Leon Takhtajan (Stony Brook University, USA)
Eisenstein–Maass series and Chern forms
on moduli spaces of curves and parabolic bundles

We show that the first Chern forms of natural Hermitian line bundles on the moduli spaces of pointed algebraic curves and stable parabolic vector bundles are explicitly expressed in terms of the special values of Eisenstein–Maass series. The cuspidal part of the curvature form of the Quillen metric in determinant line bundles associated with the families of dbar-operators on curves and on parabolic endomorphism bundles is explicitly evaluated in terms of these Chern forms.

Lin Weng (Kyushu University, Japan)
General class field theory

In this talk, we will explain our geometric approach to non-abelian Class Field Theory, initiated by Weil in his fundamental paper on Generalisation des Fonctions Abeliennes.

Motivated by the so-called Narasimhan–Seshadri correspondence in algebraic geometry, and Fontaine's theory on p-adic Galois representations in arithmetic geometry, we first introduce semi-stable filtered $(\phi, N; \omega)$-modules. Then we establish a general Class Field Theory for p-adic number fields using Tannakian categories by offering the Existence Theorem and the Reciprocity Law, based on a conjectural Micro Reciprocity Law.

Teruyoshi Yoshida (University of Cambridge, UK)
Hecke correspondences on Shimura varieties with semistable reduction

Building on T. Saito's work on the action of algebraic correspondences on the Rapoport–Zink weight spectral sequence for ℓ-adic cohomology of semistable schemes over local fields, we give an intersection-theoretic formula to compute the above action on each term of the spectral sequence. We apply this formula to compute the action of Hecke correspondences on unitary Shimura varieties of Harris–Taylor type with Iwahori level structure. This local computation is close to the spirit of Eichler–Shimura relation for modular curves.

Kōta Yoshioka (Kobe University, Japan)
Stability and the Fourier–Mukai transform

Daniel Huybrechts proved that Fourier–Mukai transform on a K3 surface preserves a nice abelian category. By using this result, we shall show that Fourier–Mukai transform preserves the stability condition, if the degree of the sheaf is sufficiently large.

Thomas Zink (Bielefeld University, Germany)
New developments in the theory of displays

Constructions in the theory of p-divisible groups as e.g. the points, the Tate module, the associated crystal, the biextensions can be constructed in an elementary way from the display of the p-divisible group. In the case where the group has an étale part this is possible by recent result of E. Lau. We also discuss an application of displays to the purity of Oort and de Jong.

Peter Zograf (Steklov Institute, Russia)
On the geometry of moduli spaces of meromorphic functions

The objective of the talk is to describe a natural Kaehler metric on the space of (generic) meromorphic functions of a given degree on compact Riemann surfaces (joint with Leon Takhtajan).